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Mosman, Karbstein: arXiv:2104.05103

# Vacuum birefringence and diffraction at XFEL: from analytical estimates to optimal parameters

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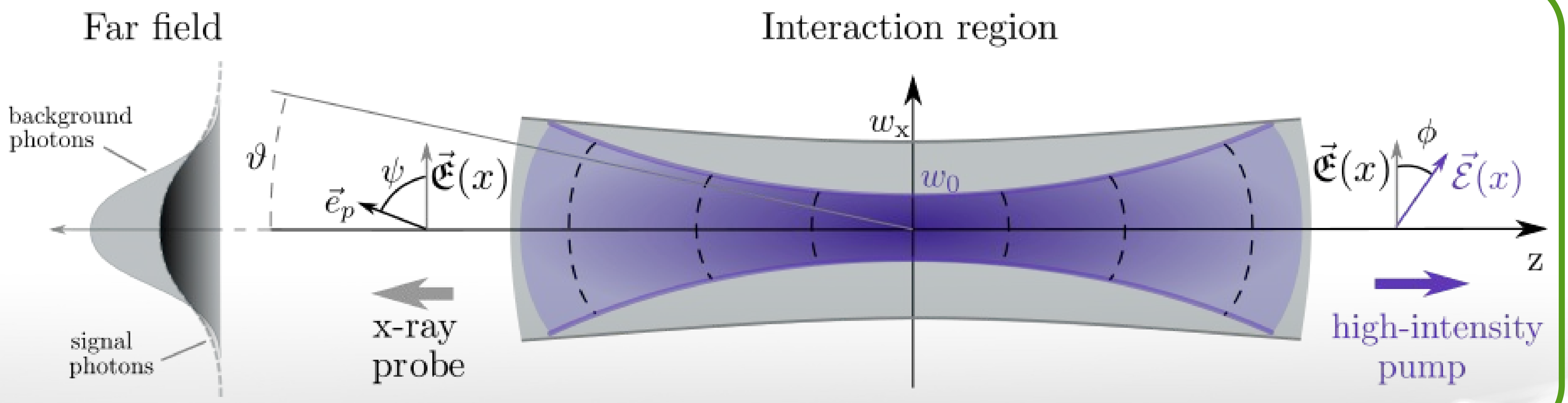
# Considered scenario



We study QED vacuum nonlinearities in the head-on collision of a loosely focused x-ray laser pulse and a tightly focused high-intensity laser beam at zero offset:




The effective vacuum-fluctuation-mediated interaction of two laser beams gives rise to signal photons mainly emitted in the forward direction of the driving beams.

# Theory



For our consideration we use the leading nonlinear correction to classical Maxwell theory

$(\hbar = c = 1)$

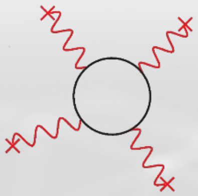
$$\mathcal{L}_{\text{HE}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overbrace{\frac{m_e^4}{360\pi^2}\left[a\,\frac{(\vec{B}^2 - \vec{E}^2)^2}{4E_{\text{cr}}^4} + b\,\frac{(\vec{B}\cdot\vec{E})^2}{E_{\text{cr}}^4}\right]} + \cdots$$

[Euler, Kockel: Naturwiss. **32** (1935)]
[Euler: Ann. Phys. **26** (1936)]

where

$$E_{\text{cr}} = \frac{m_e^2 c^3}{\hbar e} \simeq 1.3 \cdot 10^{18}\,\frac{\text{V}}{\text{m}}$$

For QED the one-loop order the term stands for:



$$a = 4, \quad b = 7$$

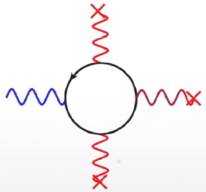
# Theory



We aim at detecting signal photons far-outside the interaction region

- start configuration: vacuum plus driving laser fields $= |0\rangle$
- signature of quantum vacuum nonlinearity is encoded in signal photons $= |\gamma_p(\vec{k})\rangle$

→ transition amplitude of “vacuum emission”

$\mathcal{S}_{(p)}(\vec{k}) = \langle \gamma_p(\vec{k})|$  $|0\rangle$

- ○ direction $\vec{k}$
- ○ energy $|\vec{k}|$
- ○ polarization $p$

[Galtsov, Skobelev: Phys. Lett. **36** (1961)]
[Karbstein, Shaisultanov: PRD **91** (2015)]

$$\mathrm{d}^3 N_{(p)}(\vec{k}) = \frac{\mathrm{d}^3 k}{(2\pi)^3} |\mathcal{S}_{(p)}(\vec{k})|^2$$

# Theory



For the driving laser fields we use:

- pulsed beam solution of the paraxial wave equation

  → valid for small asymptotic beam divergences $\theta \ll 1$

  → valid for large pulse durations $\tau \;\leftrightarrow\; \tau\omega \gg 1$

- paraxial beams fulfil $\vec{E} \perp \vec{B} \perp \vec{\kappa}$ ( $\vec{\kappa}$ direction of forward beam axis)

  and are characterized by a single field profile $\vec{E} = \mathcal{E}(x)\vec{E}\,,\;\; \vec{B} = \mathcal{E}(x)\vec{B}$

- we focus on rotationally symmetric, linearly polarized Gaussian beams

- the signal photon numbers are dominated by (quasi-)elastic contributions

[Karbstein, Gies, Reuter, Zepf: PRD **92** (2015)]

# Theory

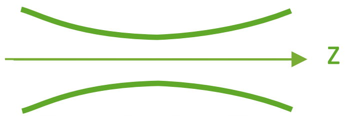


For the high-intensity laser pump we consider the following field profile

transverse profile ↓ Gouy phase ↓

$$\mathcal{E}(x) = \mathcal{E}_0 \, \mathrm{e}^{-\frac{(\mathrm{z}-t)^2}{(\tau/2)^2}} \frac{w_0}{w(\mathrm{z})} \, \mathrm{e}^{-\frac{r^2}{w^2(\mathrm{z})}} \cos\Big(\Omega(\mathrm{z}-t) + \frac{\Omega r^2}{2\,\mathrm{z}\big[1+(\frac{\mathrm{z}_R}{\mathrm{z}})^2\big]} - \arctan(\tfrac{\mathrm{z}}{\mathrm{z}_R})\Big) \,,$$

↑ pulse envelope




$$w(\mathrm{z}) = w_0\sqrt{1+(\tfrac{\mathrm{z}}{\mathrm{z}_R})^2}$$

For the x-ray probe field is justified to use in the infinite Rayleigh length approximation

$$\mathfrak{E}(x) = \mathfrak{E}_0 \, \mathrm{e}^{-\frac{(\mathrm{z}+t)^2}{(T/2)^2}} \, \mathrm{e}^{-\frac{r^2}{w_\mathrm{x}^2}} \, \cos\big(\omega(\mathrm{z}+t)\big)$$

The peak field amplitudes can be expressed as

$$\mathcal{E}_0^2 = 8\sqrt{\frac{2}{\pi}}\frac{W}{\pi w_0^2 \tau} \,, \qquad \mathfrak{E}_0^2 = 8\sqrt{\frac{2}{\pi}}\frac{N\omega}{\pi w_\mathrm{x}^2 T}$$

[Karbstein,Mosman: PRD **96** (2017)]

# Theory

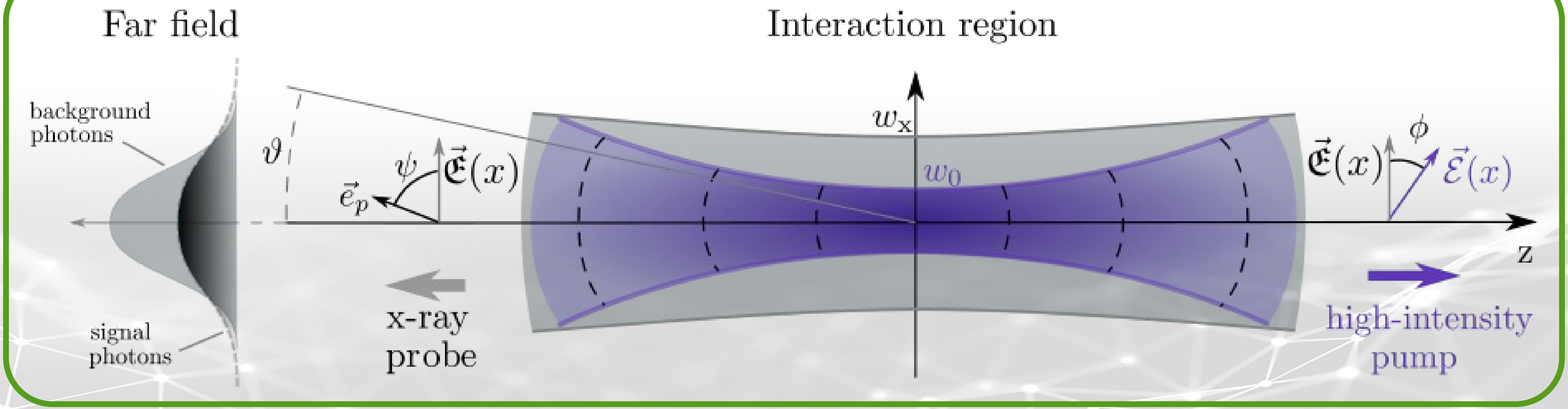


We are interested in two types of signals

→ total photon number $N_{\mathrm{tot}} = \sum_p N_{(p)}$ in polarization insensitive measurement becoming maximum for $\angle\phi = \frac{\pi}{2}$

→ polarization-flipped photons $N_{(p)} = N_{\perp}$ becoming maximum for $\angle\phi = \frac{\pi}{4}$

[Karbstein, Mosman: PRD **100** (2019)]



# Theory



The exact expression for signal photon numbers reads

$$\begin{Bmatrix} \mathrm{d}N_{\mathrm{tot}} \\ \mathrm{d}N_{\perp} \end{Bmatrix} \simeq \sin\vartheta \mathrm{d}\vartheta \begin{Bmatrix} 2(a^2+b^2)+2(a^2-b^2)\cos(2\phi) \\ (a-b)^2\sin^2(2\phi) \end{Bmatrix} \left(\frac{2}{\pi}\right)^{7/2} \frac{2\alpha^4}{45^2}(1+\cos\vartheta)^2$$
$$\times \frac{N\omega}{w_{\mathrm{x}}^2 T}\left(\frac{W}{w_0^2}\frac{\lambda_C}{m_e^3}\right)^2 \int k^3\mathrm{d}k \Bigg| \frac{1}{\sqrt{1+\frac{1}{2}(\frac{\tau}{T})^2}} \sum_{q=\pm1} \mathrm{e}^{-\frac{2\tau^2(\frac{q\omega-k}{8})^2}{1+\frac{1}{2}(\frac{\tau}{T})^2}} \int \mathrm{d}\mathrm{z} \left(\frac{w_0}{w(\mathrm{z})}\right)^2$$
$$\times \frac{1}{\left(\frac{1}{w_{\mathrm{x}}^2}+\frac{2}{w^2(\mathrm{z})}\right)} \mathrm{e}^{-\frac{(k\sin\vartheta)^2}{4\left[\frac{1}{w_{\mathrm{x}}^2}+\frac{2}{w^2(\mathrm{z})}\right]} - (\frac{4}{T})^2\frac{\mathrm{z}^2}{1+\frac{1}{2}(\frac{\tau}{T})^2} + \mathrm{i}\left[\frac{2(q\omega-k)}{1+\frac{1}{2}(\frac{\tau}{T})^2}+k(1-\cos\vartheta)\right]\mathrm{z}} \Bigg|^2$$

[Karbstein, Sundqvist: PRD **94** (2016)]
[Karbstein, Mosman: PRD **96** (2017)]

Due to its complexity this expression does not allow for systematic study of different parameter dependences of the observables

# Theory



The first attempt to simplify the expression was made by replacing the beam radius by the constant effective value

$$w(\mathrm{z}) \to w = \frac{1}{\mathrm{z}_R}\int_0^{\mathrm{z}_R} \mathrm{dz}\, w(\mathrm{z}) \simeq 1.15 w_0$$

«effective waist»

The result takes a much more compact form

$$\left\{ \begin{matrix} \mathrm{d}N_{\mathrm{tot}} \\ \mathrm{d}N_{\perp} \end{matrix} \right\} \simeq \vartheta\, \mathrm{d}\vartheta \left\{ \begin{matrix} 2(a^2+b^2) + 2(a^2-b^2)\cos(2\phi) \\ (a-b)^2 \sin^2(2\phi) \end{matrix} \right\} \frac{4\alpha^4}{225(3\pi)^{\frac{3}{2}}} N \left(\frac{W}{m_e}\frac{\lambda_{\mathrm{C}}}{w_0}\right)^2 \left(\frac{\omega}{m_e}\right)^4$$

$$\times \frac{\left(\frac{w_{\mathrm{x}}}{w}\right)^2}{\left[1+2\left(\frac{w_{\mathrm{x}}}{w}\right)^2\right]^2} \mathrm{e}^{-\frac{1}{2}\frac{(\omega\vartheta w_{\mathrm{x}})^2}{1+2\left(\frac{w_{\mathrm{x}}}{w}\right)^2}} F\left(\frac{4\mathrm{z}_R}{\sqrt{T^2+\frac{1}{2}\tau^2}}, \frac{T}{\tau}\right)$$

[Karbstein: PRD **98** (2018)]

The remaining integral is a “simple” function with known properties

$$F(\chi,\rho) := \sqrt{\frac{1+2\rho^2}{3}}\, \chi^2\, \mathrm{e}^{2\chi^2} \int \mathrm{d}\kappa\, \mathrm{e}^{-\kappa^2} \left| \sum_{\ell=\pm 1} \mathrm{e}^{2\ell\rho\kappa\chi} \mathrm{erfc}(\ell\rho\kappa + \chi) \right|^2$$

# Theory



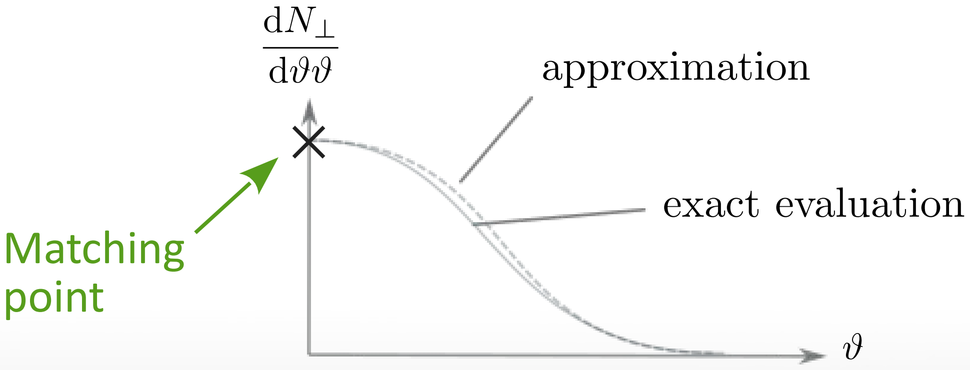
The approximation is not accurate everywhere. It especially worsens for larger ratios $\beta = w_{\mathrm{x}}/w_0 \geq 1$. This makes it inappropriate for quantitative study of signal photons

| $w_{\mathrm{x}}/w_0$ | $\lvert 1 - N_\perp/N_\perp^{\mathrm{full}} \rvert$ |
|---|---|
| 1/10 | 2.6% |
| 1/3 | 0.2% |
| 1 | 8.0% |
| 3 | 13.8% |

[Karbstein: PRD **98** (2018)]

# Theory

Here we use a more advanced strategy to fix the effective waist, namely we choose it such that for $\vartheta = 0$ the approximate result for the differential number of signal photons matches the exact result for this value




In this case effective waist is determined by geometric properties: it is a function of the pulse durations $\tau$, $T$ and waist sizes $w_x$, $w_0$ of both the pump and the probe beams as well as the Rayleigh length of the pump $z_R$

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# Theory

New approximation improves accuracy → deviations <1%

| $w_x/w_0$ | $\lvert 1 - N_\perp/N_\perp^{\rm full}\rvert$ | |
|---|---|---|
| | $w \simeq 1.15 w_0$ | new approximation |
| 1/10 | 2.6% | 0.9% |
| 1/3 | 0.2% | 0.9% |
| 1 | 8.0% | 0.5% |
| 3 | 13.8% | 0.1% |

Allows to use analytic methods for finding the parameters optimising the signal.

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# Parameters

- For our study we choose the 300TW Relativistic Laser at XFEL (ReLaX):

$$W = 10\text{J}\,,\ \tau^{FWHM} = 25\text{fs}\,,\ \lambda = 800\text{nm}\,,\ w_0^{HWHM} = 1\mu\text{m}\,,\ 1\text{Hz}\,.$$

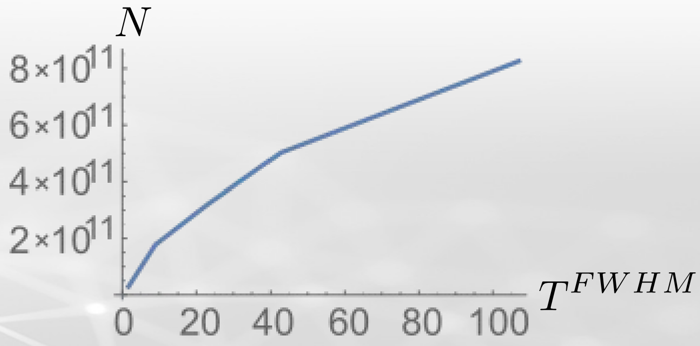
- For the probe beam we employ the parameters available at European XFEL at the frequency $\omega = 12914\text{eV}$: $T^{FWHM} \simeq 1.67\ldots107\text{fs}$

[Technical Report, European XFEL(2011)]

- For the polarisation purity we use $\mathcal{P} \simeq 1.4\times10^{-11}$, achieved with silicon channel-cut polarizer (four reflections)

[Marx, et al., PRL **110** (2013) +private communications 2021]

- The amount of probe photons $N$ in x-ray beam depends on the frequency and pulse time

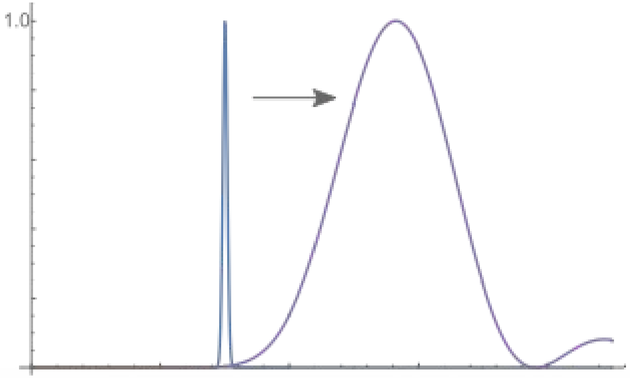



- We treat the waist beam size $w_x$ as a free parameter

# Parameters

The original probe pulse duration will be enlarged by the silicon polarizer.

[Lindberg, Shvyd'ko, Phys. Rev. ST Accel.Beams **15**, (2012)]
[Shvyd'ko, Lindberg, Phys. Rev. ST Accel.Beams **15**, (2012)]




$$T \to T_{\mathcal{P}}$$

«incident» «interaction region»

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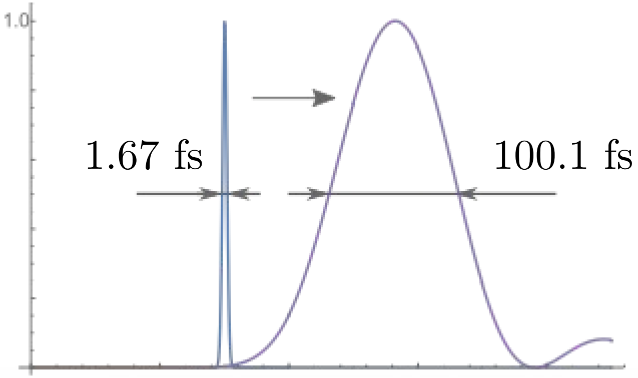



$$T \rightarrow T_{\mathcal{P}}$$

«incident» «interaction region»

Hence we have the following dependence between photon number and probe pulse duration

| # | $T^{FWHM}$ [fs] | $T_{\mathcal{P}}^{FWHM}$ [fs] | $N$ |
|---|---|---|---|
| (1) | 1.67 | 100.1 | $2.98 \times 10^{10}$ |
| (2) | 8.96 | 100.3 | $1.78 \times 10^{11}$ |
| (3) | 23.2 | 101.4 | $3.22 \times 10^{11}$ |
| (4) | 42.8 | 104.5 | $5.03 \times 10^{11}$ |
| (5) | 107 | 129 | $8.26 \times 10^{11}$ |

# Results

We analyse the parameter dependence of the signal accessible by three different “observables”:

- the integrated number of polarization-flipped signal photons $N_{\perp}$
- the discernible number of polarization-flipped signal photons $N_{\perp>}$
- and the total number of discernible signal photons attainable in a polarization insensitive measurement $N_{\mathrm{tot}>}$

# Results: integrated number of $N_\perp$



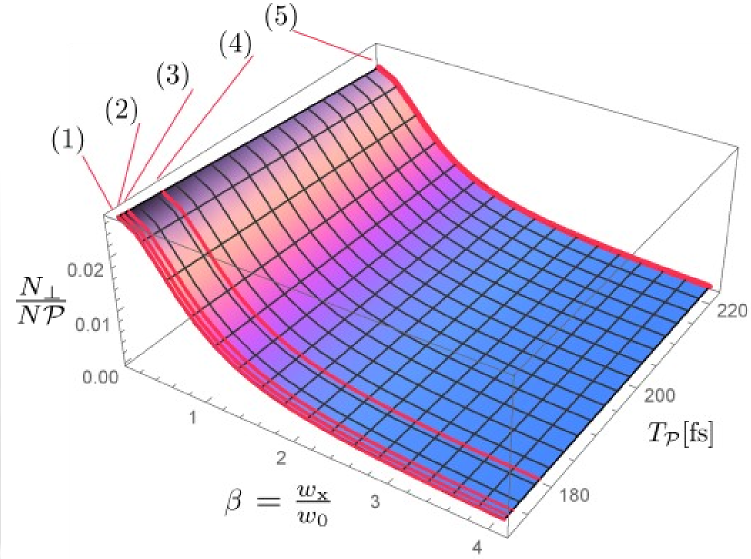
For the integrated number of polarization-flipped signal photons $N_\perp$ we get the following compact expression

$$\frac{N_\perp}{\mathcal{P}N} \simeq \frac{4\alpha^4}{25(3\pi)^{\frac{3}{2}}} \left(\frac{W}{m_e}\frac{\omega}{m_e}\right)^2 \left(\frac{\lambda\!\!\!^-_C}{w_0}\right)^4 \frac{1}{1+2\beta^2} \frac{\sqrt{F_\beta F_0}}{\mathcal{P}}$$

Here we use the shorthand notations

$$F_\beta := F\left(\frac{4z_R\sqrt{1+2\beta^2}}{\sqrt{T^2+\frac{1}{2}\tau^2}}, \frac{T}{\tau}\right) \quad \text{and} \quad \beta := \frac{w_x}{w_0}$$

We can analyse now the dependence on the parameters $w_x$ and $T_{\mathcal{P}}^{FWHM}$



# Results: integrated number of $N_\perp$



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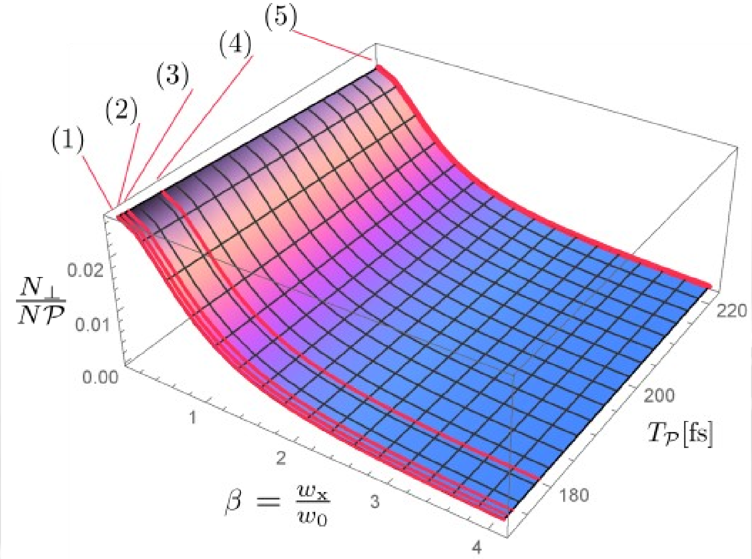
Here we use the shorthand notations

$$F_\beta := F\left(\frac{4z_R\sqrt{1+2\beta^2}}{\sqrt{T^2+\frac{1}{2}\tau^2}}, \frac{T}{\tau}\right) \quad \text{and} \quad \beta := \frac{w_x}{w_0}$$

We can analyse now the dependence on the parameters $w_x$ and $T_{\mathcal{P}}^{FWHM}$



# Results: integrated number of $N_\perp$



In the limit $\beta \ll 1$ we have the following compact expression for the integrated number of polarization-flipped signal photons

$$\frac{N_\perp}{\mathcal{P}N} \simeq \frac{4\alpha^4}{25(3\pi)^{\frac{3}{2}}}\left(\frac{W}{m_e}\frac{\omega}{m_e}\right)^2\left(\frac{\lambdabar_C}{w_0}\right)^4\frac{F_0}{\mathcal{P}}$$

[Karbstein: PRD **98** (2018)]

From it we can obtain the **discernibility criterion**

$$\frac{4\alpha^4}{25(3\pi)^{\frac{3}{2}}}\left(\frac{W}{m_e}\frac{\omega}{m_e}\right)^2\left(\frac{\lambdabar_C}{w_0}\right)^4\frac{F_0}{\mathcal{P}} = 1$$

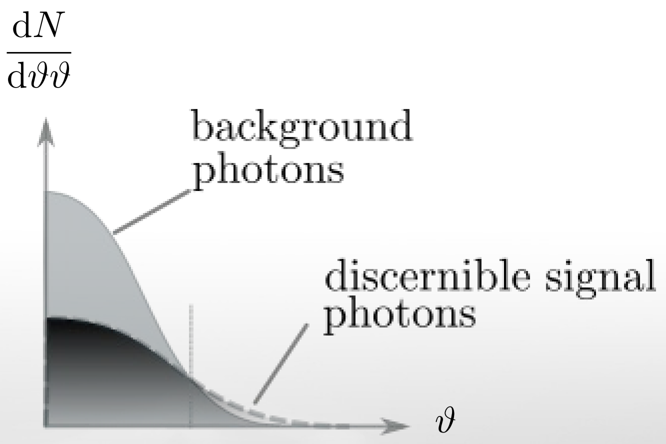
For the current HiBEF parameters and polarization purity $\mathcal{P} = 1.4 \times 10^{-11}$ the maximal achievable value is 1/40

# Results: discernible number of $N_\perp$



The discernible number of polarization-flipped signal photons $N_{\perp>}$ is defined by

$$\frac{\mathrm{d}N_\perp}{\vartheta\,\mathrm{d}\vartheta} \geq \mathcal{P}\,\frac{\mathrm{d}N}{\vartheta\,\mathrm{d}\vartheta}$$



# Results: discernible number of $N_\perp$

The discernible number of polarization-flipped signal photons $N_{\perp >}$ is defined by

$$\frac{\mathrm{d}N_\perp}{\vartheta\,\mathrm{d}\vartheta} \geq \mathcal{P}\,\frac{\mathrm{d}N}{\vartheta\,\mathrm{d}\vartheta}$$

and is given by the slightly more complicated formula

$$N_{\perp >} \simeq \mathcal{P}N\,(1+2\beta^2)\sqrt{\frac{F_0}{F_\beta}}\left(\frac{4\alpha^4}{25(3\pi)^{\frac{3}{2}}}\left(\frac{W}{m_e}\,\frac{\omega}{m_e}\right)^2\left(\frac{\lambda\!\!\!{}^-_{\mathrm{C}}}{w_0}\right)^4\frac{F_\beta/\mathcal{P}}{(1+2\beta^2)^2}\right)^{\frac{1}{1-\frac{1}{1+2\beta^2}\sqrt{\frac{F_\beta}{F_0}}}}$$

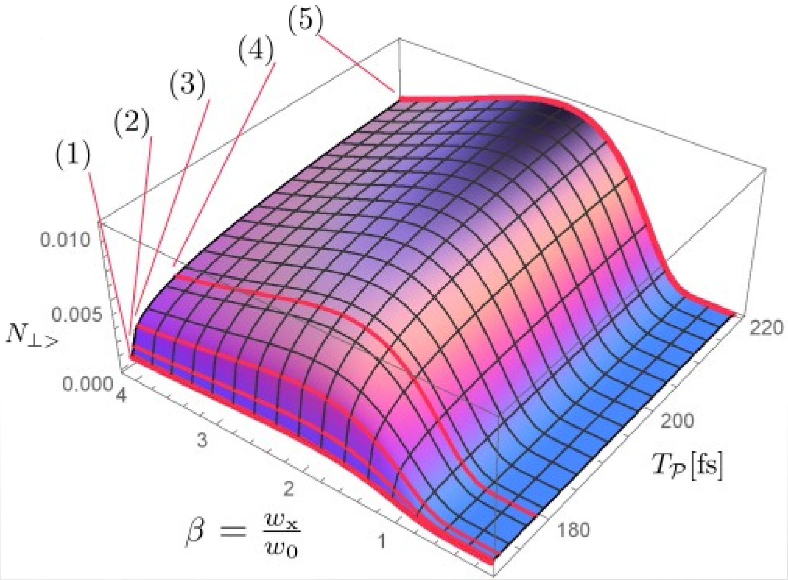
Were

$$F_\beta := F\left(\frac{4\mathrm{z}_R\sqrt{1+2\beta^2}}{\sqrt{T^2+\frac{1}{2}\tau^2}},\frac{T}{\tau}\right) \quad \text{and} \quad \beta := \frac{w_{\mathrm{x}}}{w_0}$$

# Results: discernible number of $N_\perp$



The dependence of the discernible photon number $N_{\perp >}$ on parameters $w_{\rm x}$ and $T_{\mathcal{P}}^{FWHM}$ is more complicated




- optimal time for measuring is maximum
- at current parameters of experiment there is negligible amount of discernible photons at waist $\beta = w_{\rm x}/w_0 \leq 1$
- there is an optimal waist $\beta > 1$ maximizing discernible signal $N_{\perp >}$
- For the HiBEF parameters the maximum number of discernible photos is $N_{\perp >} \simeq 0.01/\mathrm{shot}$ at $w_{\rm x} = 2.1 w_0$

# Results: discernible number of $N_\perp$



The optimal waist for this observable depends on the other parameters of experiment and can be explicitly found from the equation

$$\frac{4\alpha^4}{25(3\pi)^{\frac{3}{2}}}\left(\frac{W}{m_e}\frac{\omega}{m_e}\right)^2\left(\frac{\lambda\!\!\!^{-}_{\mathrm{C}}}{w_0}\right)^4\frac{F_0}{\mathcal{P}}=\frac{1}{\chi^2}\exp\left(\chi-\frac{1}{\chi}\right)$$

where

$$\chi=\frac{1}{1+2\beta^2}\sqrt{\frac{F_\beta}{F_0}}\Bigg|_{\beta=\beta_{\mathrm{opt}}}\in\ (0,1]$$

The solution of the equation for optimal waist exists when the l.h.s is < 1. In the opposite case all the photons become discernible at waist $\beta\ll1$

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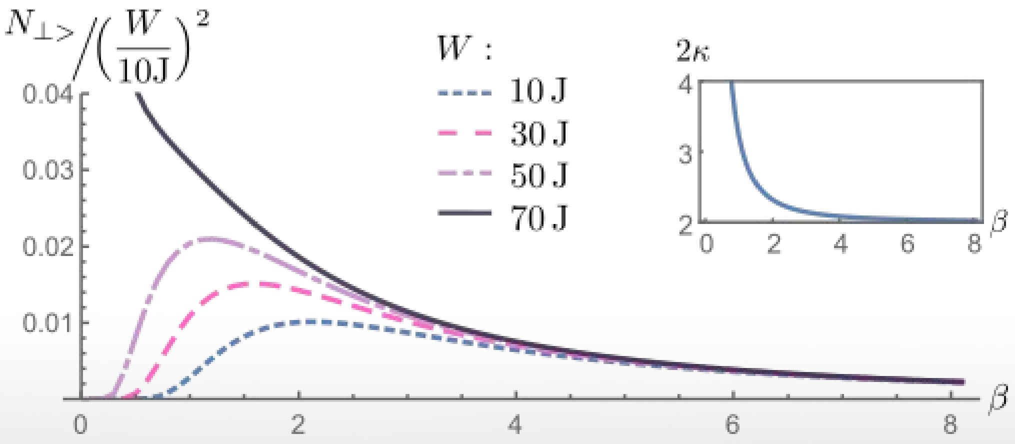
# Results: discernible number of $N_\perp$

Let us demonstrate the dependence on the optimal waist on the other parameters of experiment




The dependence of **normalized** discernible signal on the energy is shown in the figure.

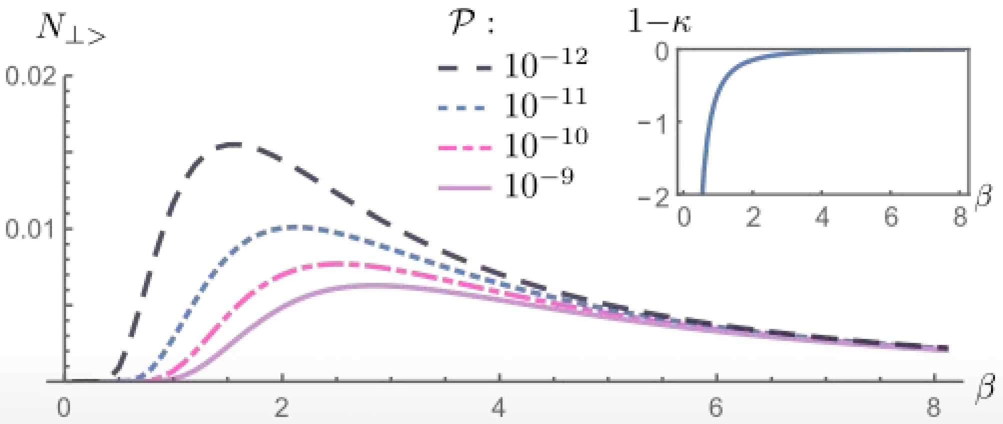
At waist $w_{\mathrm{x}} = 2.1 w_0$ small changes of energy will change the signal as

$$N_{\perp>} \sim W^{2\kappa} \simeq W^{2.3}$$

# Results: discernible number of $N_\perp$



Let us demonstrate the dependence on the optimal waist on the other parameters of experiment




The dependence of discernible signal on the polarisation purity is shown on the figure.

At waist $w_x = 2.1 w_0$ small changes of energy will change the signal as $N_{\perp>} \sim \mathcal{P}^{1-\kappa} \simeq \mathcal{P}^{-0.14}$

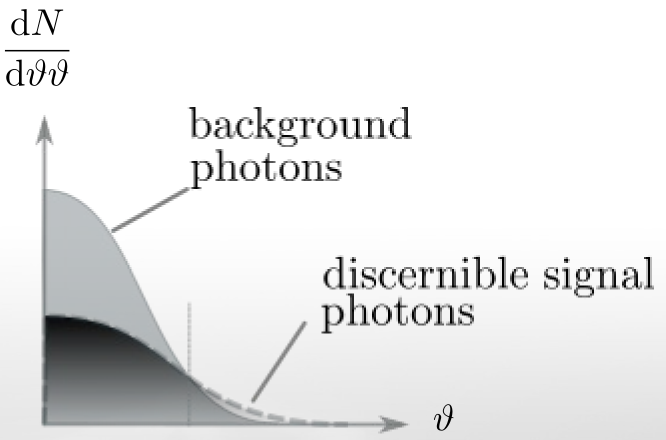
The ratio for the number of signal photons per background photon scattered outside the discernibility angle

$$\frac{N_{\perp>}}{N_> \mathcal{P}} \simeq 8$$

# Results: discernible number of $N_{\mathrm{tot}}$

The discernible number of total signal photons $N_{\mathrm{tot}>}$ is defined by

$$\frac{\mathrm{d}N_{\mathrm{tot}}}{\vartheta\,\mathrm{d}\vartheta} \geq \frac{\mathrm{d}N}{\vartheta\,\mathrm{d}\vartheta}$$



# Results: discernible number of $N_{\mathrm{tot}}$

The discernible number of total signal photons $N_{\mathrm{tot}>}$ is defined by

$$\frac{\mathrm{d}N_{\mathrm{tot}}}{\vartheta\,\mathrm{d}\vartheta} \geq \frac{\mathrm{d}N}{\vartheta\,\mathrm{d}\vartheta}$$

and is given by

$$N_{\mathrm{tot}>} \simeq N\,(1+2\beta^2)\sqrt{\frac{F_0}{F_\beta}}\left(\frac{784\alpha^4}{225(3\pi)^{\frac{3}{2}}}\left(\frac{W}{m_e}\frac{\omega}{m_e}\right)^2\left(\frac{\lambda\!\!\!^{-}_{\mathrm{C}}}{w_0}\right)^4\frac{F_\beta}{(1+2\beta^2)^2}\right)^{\frac{1}{1-\frac{1}{1+2\beta^2}\sqrt{\frac{F_\beta}{F_0}}}}$$

Were

$$F_\beta := F\left(\frac{4z_R\sqrt{1+2\beta^2}}{\sqrt{T^2+\frac{1}{2}\tau^2}}, \frac{T}{\tau}\right) \quad \text{and} \quad \beta := \frac{w_{\mathrm{x}}}{w_0}$$

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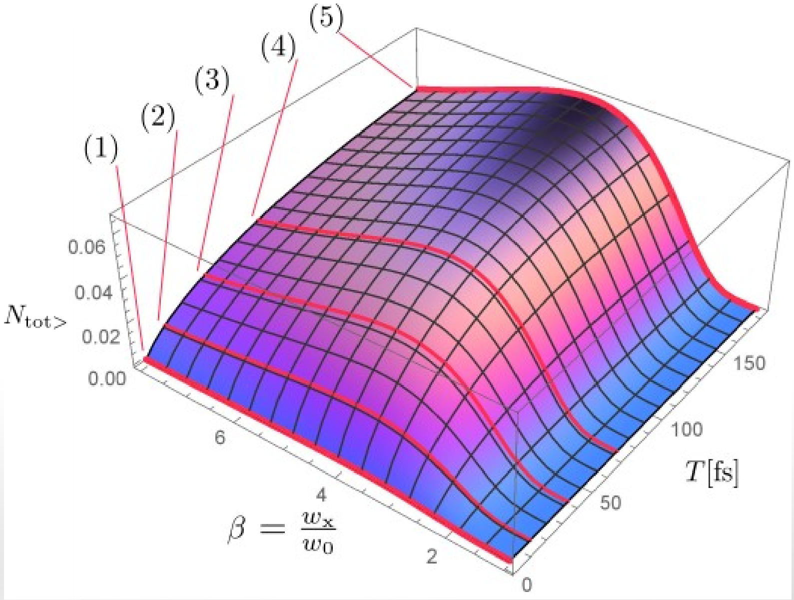
# Results: discernible number of $N_{\rm tot}$

The dependence of the discernible photon number $N_{\rm tot>}$ on parameters $w_{\rm x}$ and $T^{FWHM}$ is more complicated

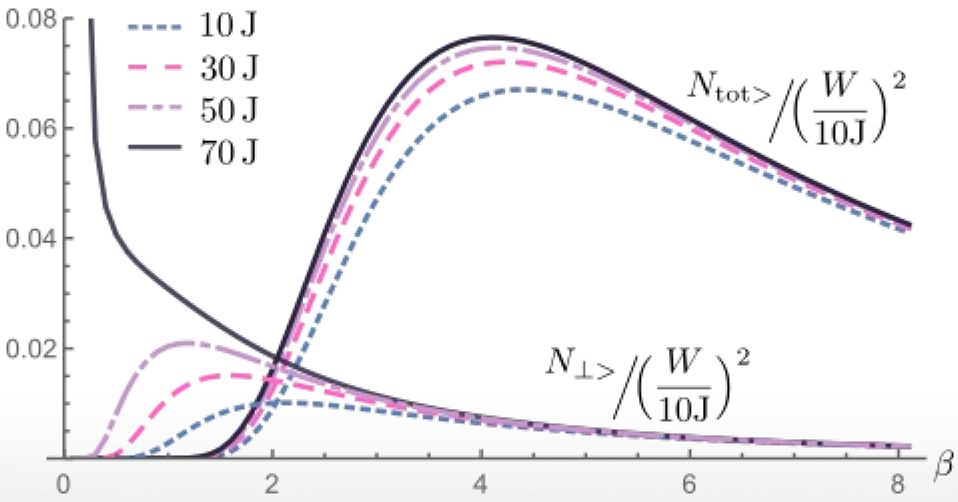



- As there no polarizer is not needed, we use the original pulse durations of the XFEL. The optimal duration for measuring again is the maximum one
- there is an optimal waist maximizing discernible signal $N_{\rm tot>}$
- For current HiBEF parameters the maximum number of discernible photons is $N_{\rm tot>} \simeq 0.07/{\rm shot}$ at $w_{\rm x} = 4.5 w_0$

# Results: discernible number of $N_{\mathrm{tot}}$



Let us demonstrate the dependence of optimal waist on the other parameters of experiment




The dependence of the **normalized** discernible signal on the energy is shown on the figure.

At waist $w_{\mathrm{x}} = 4.5 w_0$ small changes of energy will change the signal as

$$N_{\mathrm{tot}>} \sim W^{2\kappa} \simeq W^{2.06}$$

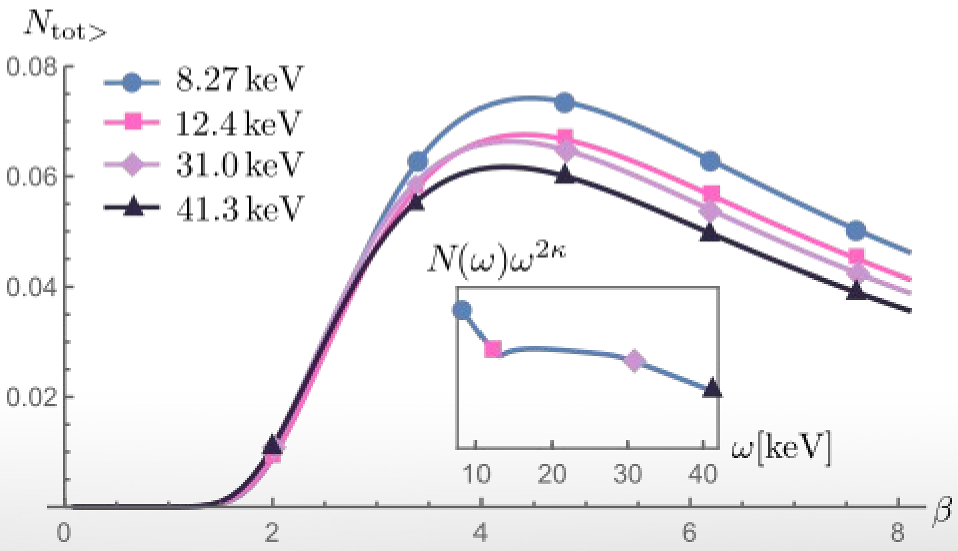
The ratio for the number of signal photons per background photon scattered outside the discernibility angle

$$\frac{N_{\mathrm{tot}>}}{N_{>}} \simeq 32.6$$

# Results: discernible number of $N_{tot}$



For this observable we can also use different probe photon energies




The probe photon number at XFEL raises with decrease of energy. A smaller energy may increase the effect

$$N_{tot>} \sim N(\omega)\omega^{2.06}$$

# Conclusions



Let us come back to the original expression for signal photons

$$\begin{Bmatrix} \mathrm{d}N_{\mathrm{tot}} \\ \mathrm{d}N_{\perp} \end{Bmatrix} \simeq \vartheta\,\mathrm{d}\vartheta \begin{Bmatrix} 2(a^2+b^2)+2(a^2-b^2)\cos(2\phi) \\ (a-b)^2\sin^2(2\phi) \end{Bmatrix} \frac{4\alpha^4}{225(3\pi)^{\frac{3}{2}}} N\left(\frac{W}{m_e}\frac{\lambda\!\!\!{}^{-}_{\mathrm{C}}}{w_0}\right)^2\left(\frac{\omega}{m_e}\right)^4$$

$$\times \frac{(\frac{w_{\mathrm{x}}}{w})^2}{[1+2(\frac{w_{\mathrm{x}}}{w})^2]^2}\,\mathrm{e}^{-\frac{1}{2}\frac{(\omega\vartheta w_{\mathrm{x}})^2}{1+2(\frac{w_{\mathrm{x}}}{w})^2}}\,F\left(\frac{4z_R}{\sqrt{T^2+\frac{1}{2}\tau^2}},\frac{T}{\tau}\right)$$

# Conclusions



Let us come back to the original expression for signal photons

$$\left\{\begin{matrix} \mathrm{d}N_{\mathrm{tot}} \\ \mathrm{d}N_{\perp} \end{matrix}\right\} \simeq \vartheta\, \mathrm{d}\vartheta \left\{\begin{matrix} 2(a^2+b^2)+2(a^2-b^2)\cos(2\phi) \\ (a-b)^2 \sin^2(2\phi) \end{matrix}\right\} \frac{4\alpha^4}{225(3\pi)^{\frac{3}{2}}} N \left(\frac{W}{m_e}\frac{\lambda\!\!\!^{-}_{\mathrm{C}}}{w_0}\right)^2 \left(\frac{\omega}{m_e}\right)^4$$

$$\times \frac{\left(\frac{w_{\mathrm{x}}}{w}\right)^2}{[1+2(\frac{w_{\mathrm{x}}}{w})^2]^2} \mathrm{e}^{-\frac{1}{2}\frac{(\omega\vartheta w_{\mathrm{x}})^2}{1+2(\frac{w_{\mathrm{x}}}{w})^2}} F\left(\frac{4 z_R}{\sqrt{T^2+\frac{1}{2}\tau^2}}, \frac{T}{\tau}\right)$$

Measurement of both signals (or total number at different polarization angles) allows to identify separately coefficients *a* and *b* in the Heisenberg-Euler action

$$\mathcal{L}_{\mathrm{HE}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m_e^4}{360\pi^2}\left[a\,\frac{(\vec{B}^2-\vec{E}^2)^2}{4E_{\mathrm{cr}}^4} + b\,\frac{(\vec{B}\cdot\vec{E})^2}{E_{\mathrm{cr}}^4}\right] + \cdots$$

These coefficients encode the information about higher-loop corrections and, potentially, of new physics.

# Conclusions



- We derived compact analytic approximation, allowing to predict signal photon numbers with high accuracy
- Using the approximation we identified the optimal parameters for three different observables:

→ $\frac{N_\perp}{\mathcal{P}N}$ is optimized at smallest pulse duration $T$ and probe waist $w_{\mathrm{x}}$

→ $\mathrm{N}_{\perp>}$ at largest pulse duration $T$ and waist $w_{\mathrm{x}}$

→ $N_{\mathrm{tot}>}$ at largest pulse duration $T$ and waist $w_{\mathrm{x}}$ and smallest photon energy

# THANK YOU!

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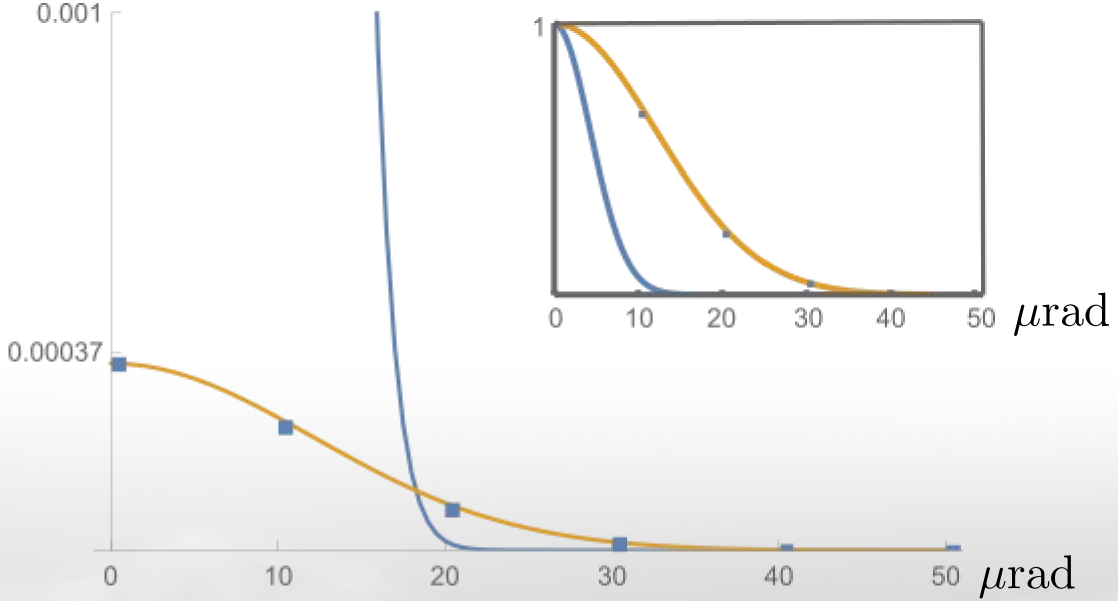
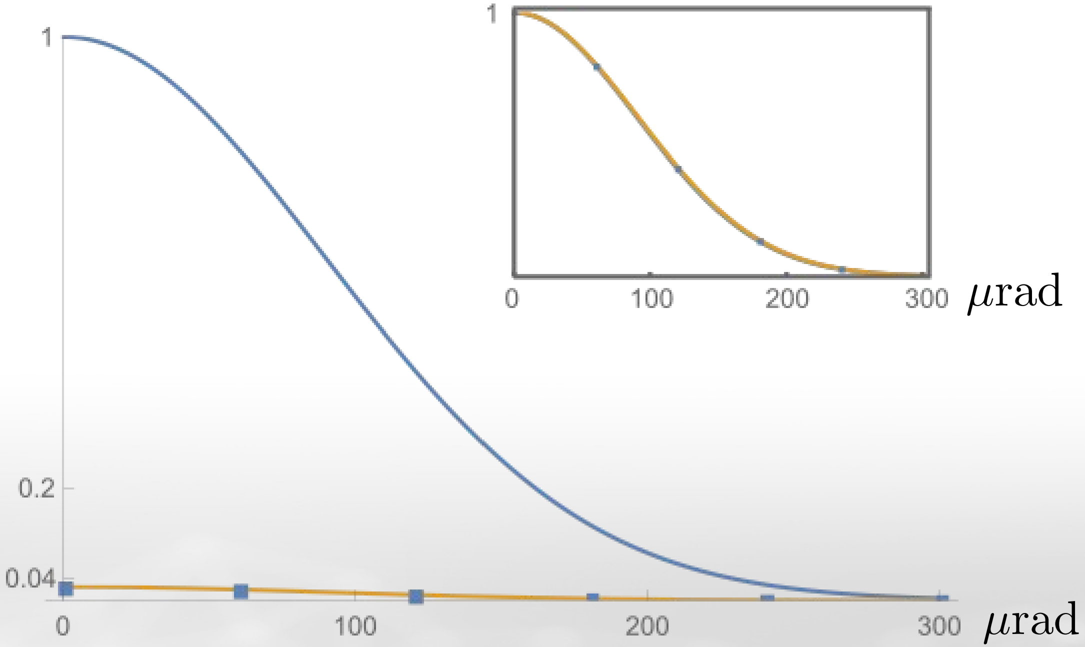
# Appendix

Angular distribution for polarisation-flipped signal photons for HiBEF parameters and $w_x = 2.1 w_0$



# Appendix

Angular distribution for polarisation-flipped signal photons for HiBEF parameters and
$w_{\mathrm{x}} = 0.1 w_0$



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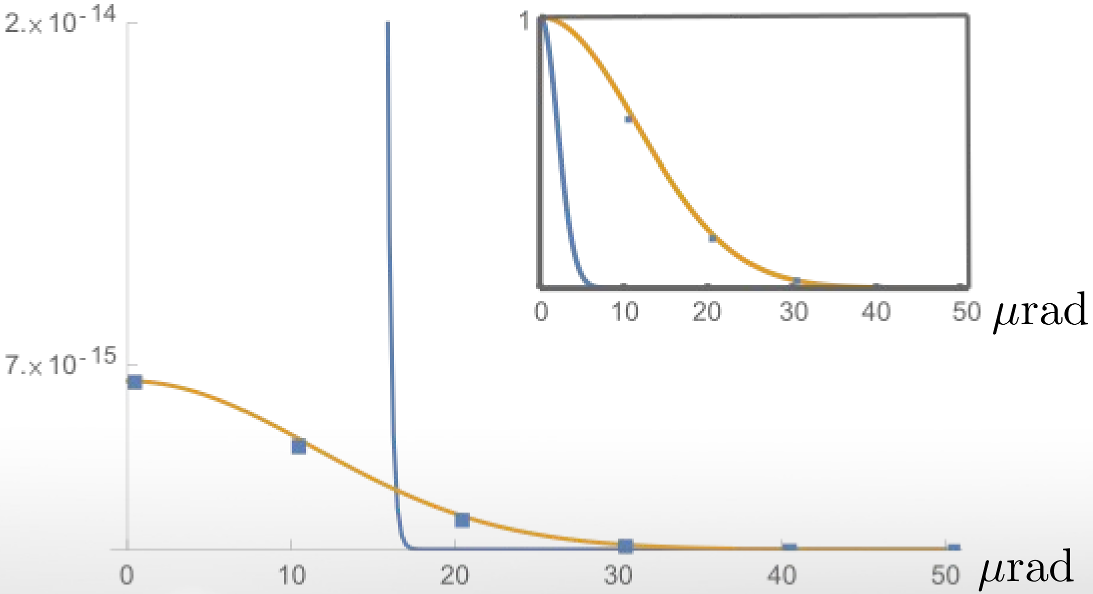
# Appendix

Angular distribution for total signal photons for HiBEF parameters and $w_{\mathrm{x}} = 4.5w_0$



# Appendix



Estimation for the amount of discernible signal photons $N_{\perp>}$ for HiBEF parameters