Based on work done at Yale:
[**KN**, S. Pirandola, L. Jiang, Nature Communications, **11**, 457 (2020)]

Enhanced energy-constrained quantum communication over bosonic Gaussian channels using multi-channel strategies

Kyungjoo Noh aws center for quantum computing

QIP 2021 (2021/02/01)

Problem formulation (setup)

Quantum communication over noisy quantum channels

$\mathcal{N}_{A\to B}$ noisy channel

$\hat{\rho}_A \longrightarrow$  $\longrightarrow \hat{\sigma}_B = \mathcal{N}_{A\to B}(\hat{\rho}_A)$

input state

noisy output state

energy attenuation

Question:

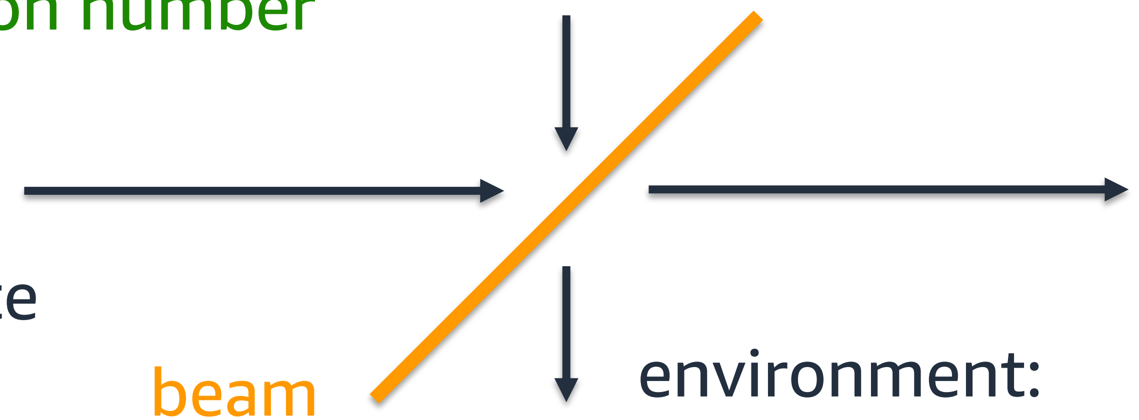
How many qubits can be faithfully transmitted through the noisy channel per channel use? or what is the quantum capacity of the noisy channel?

some obvious answers (in the unit of qubit per channel use):

- $= 1$ for ideal qubit channels
- < 1 for noisy qubit channels (due to overheads associated with QEC or ED)
- $\rightarrow \infty$ for ideal bosonic channels (infinite dimensionality of bosonic modes)

Problem formulation (setup)

Quantum communication over **lossy bosonic channels**



$\eta = 1$: perfect transmission (or $\gamma = 0$)

$\eta = 0$: complete loss of information to the environment (or $\gamma = 1$)

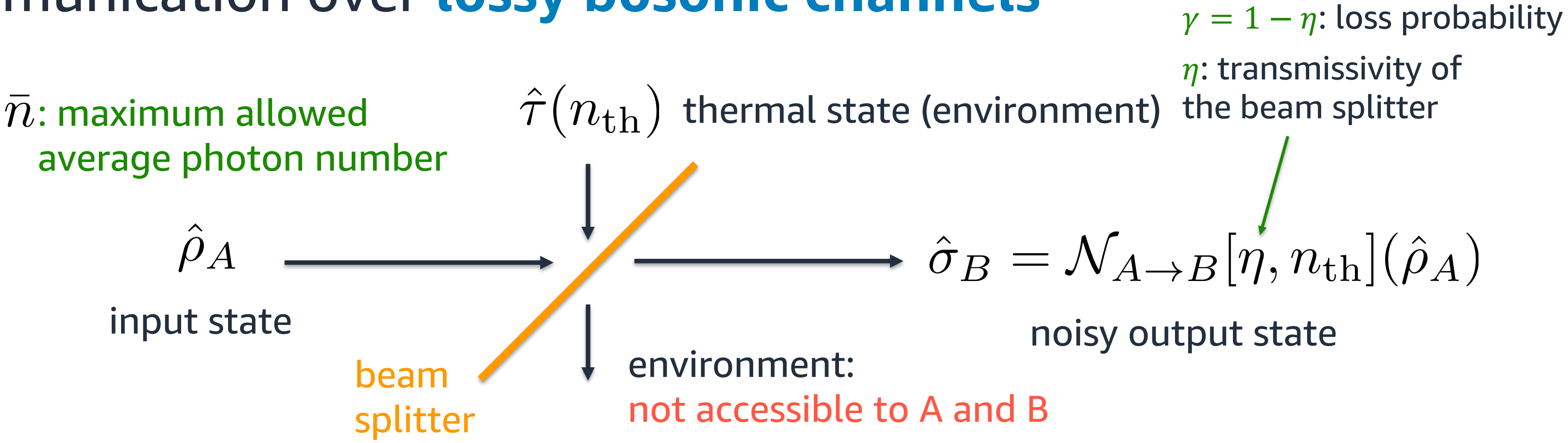
$n_{th} = 0$: vacuum environment (optical frequency) → pure-loss channel

$n_{th} > 0$: thermal environment (microwave frequency) → thermal-loss channel

because microwave frequency is typically not low enough compared to the environmental temperature

Problem formulation (known results; $\bar{n} \to \infty$)

Quantum communication over lossy bosonic channels



known result for the pure-loss channel
($n_{th} = 0$; energy-unconstrained case, i.e., $\bar{n} \to \infty$)

A. S. Holevo and R. F. Werner, PRA **63**, 032312 (2001)
M. M. Wolf, D. Pérez-García, and G. Giedke, PRL **98**, 130501 (2007)
see also, e.g., IEEE Trans. Info. Theory **64**, 7802–7827 (2018) and IEEE Trans. Info. Theory **65**, 2563–2582 (2019),

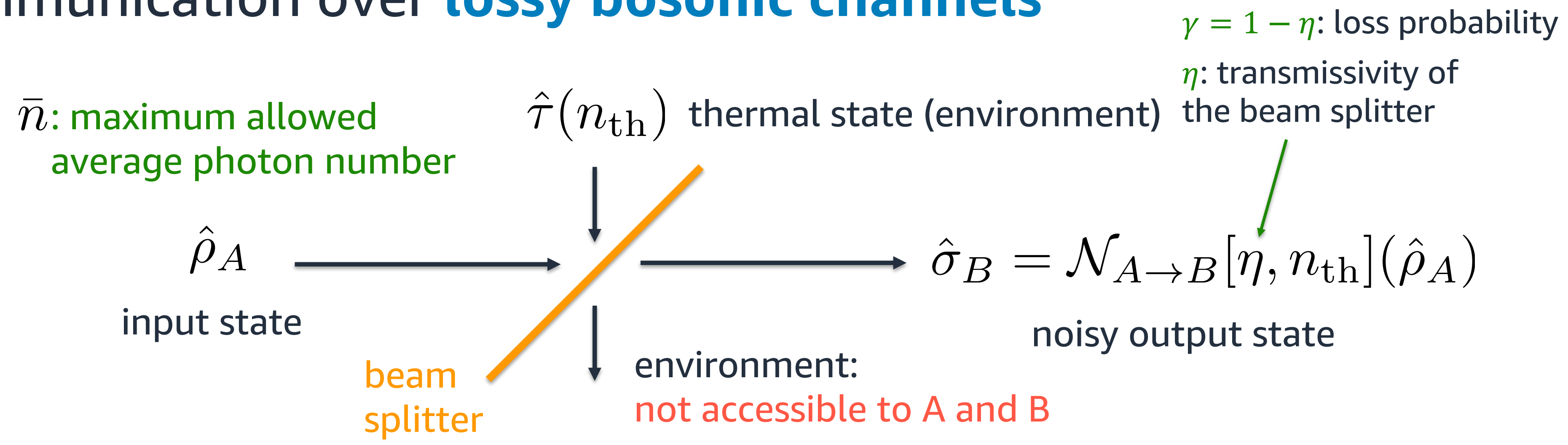
$$C_Q(\mathcal{N}[\eta, 0]) = \log_2\left(\frac{\eta}{1-\eta}\right)$$

$C_Q \to \infty$ as $\eta \to 1$ (or $\gamma \to 0$):
as expected

$C_Q = 0$ for any $\eta \leq 0.5$ (or $\gamma \geq 0.5$):
consistent with the no-cloning theorem

Problem formulation (known results; $\bar{n} < \infty$)

Quantum communication over **lossy bosonic channels**



known result for the pure-loss channel
($n_{th} = 0$; energy-constrained case, i.e., $\bar{n} < \infty$)

M. M. Wilde, H. Qi, IEEE Trans. Info. Theory **64**, 7802–7827 (2018)

$$C_Q^{\leq \bar{n}}(\mathcal{N}[\eta, n_{\text{th}}]) = g(\eta \bar{n}) - g((1-\eta)\bar{n})$$

where

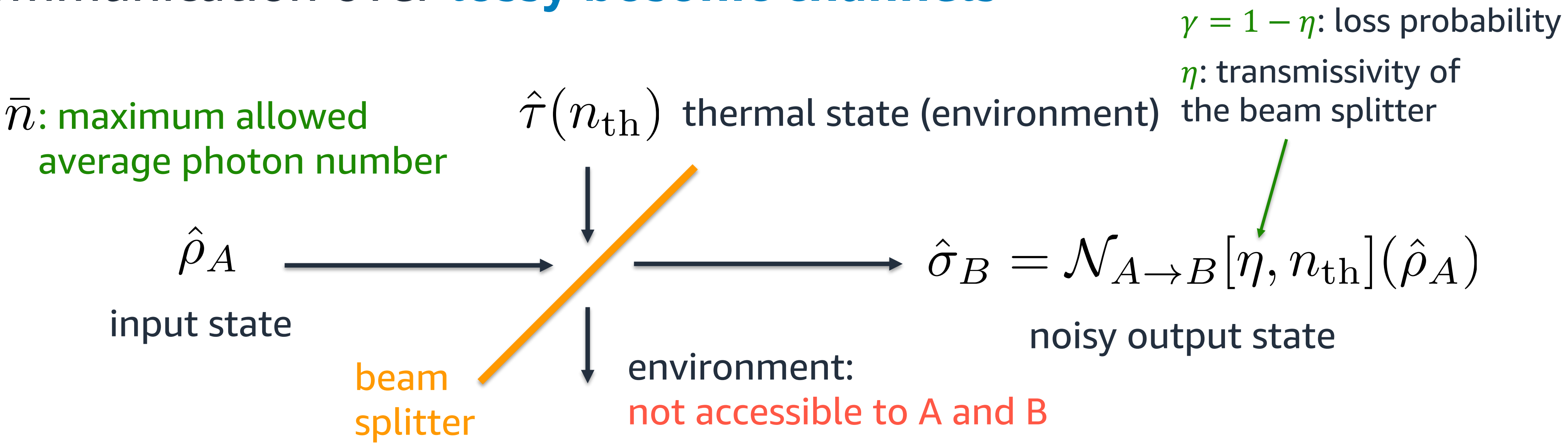
$$g(x) \equiv (x+1)\log_2(x+1) - x \log_2 x$$

$C_Q \to g(\bar{n})$ " $< \infty$" at $\eta = 1$ (or $\gamma = 0$):
entropy of a thermal state with an average photon number $\bar{n}$

$C_Q = 0$ for any $\eta \leq 0.5$ (or $\gamma \geq 0.5$):
still consistent with the no-cloning theorem

Problem formulation

Quantum communication over **lossy bosonic channels**



Problem:

What is the quantum capacity of a thermal-loss channel ($n_{th} > 0$) in the energy-constrained ($\bar{n} < \infty$) scenario?

Result:

KN, S. Pirandola, L. Jiang, Nature Communications, **11**, 457 (2020)

Discovered a multi-channel strategy which outperforms the best known single-channel strategy (hence establishing the tightest lower bound of the energy-constrained Gaussian quantum channel capacity)

Coherent information and quantum capacity

Coherent information and quantum capacity

coherent information — B. Schumacher and M. A. Nielsen, PRA **54**, 2629 (1996)

$$I_c(\mathcal{N}, \hat{\rho}) \equiv S(\mathcal{N}(\hat{\rho})) - S(\mathcal{N}^c(\hat{\rho}))$$

von Neumann entropy — complementary channel

maximum obtainable entanglement via one-way entanglement distillation using the channel $\mathcal{N}$ and (a purification of) the state $\hat{\rho}$

S. Lloyd, PRA **55**, 1613 (1997)
I. Devetak, IEEE Trans. Inf. Theory **51**, 44–55 (2005)

regularized coherent information

$$C_Q(\mathcal{N}) = Q_{\mathrm{reg}}(\mathcal{N}) \equiv \lim_{N\to\infty} \frac{1}{N} \max_{\hat{\rho}} I_c(\mathcal{N}^{\otimes N}, \hat{\rho})$$

quantum capacity

the need for regularization ($N \to \infty$) makes it very challenging to evaluate the quantum capacity of a generic channel

$N = 1$ suffices for degradable channels (e.g., pure-loss channels)

Coherent information and quantum capacity

KN, S. Pirandola, L. Jiang, Nature Communications, **11**, 457 (2020)

	pure-loss ($n_{th} = 0$)	thermal-loss ($n_{th} > 0$)
Energy-constrained ($\bar{n} < \infty$)	single-mode thermal state $\hat{\tau}(\bar{n})$ is optimal	multi-mode correlated thermal states sometimes outperform single-mode thermal states (our result)
Energy-unconstrained ($\bar{n} \to \infty$)	single-mode thermal state $\hat{\tau}(\bar{n} \to \infty)$ (of infinite temperature) is optimal	single-mode thermal state $\hat{\tau}(\bar{n} \to \infty)$ remains to be the best known input state

↑ degradable, single-channel strategy is optimal

↑ not degradable, single-channel strategy may not be optimal

An example for inspiration

Qubit depolarization channel

$$\mathcal{N}_D[p](\hat{\rho}) = (1-p)\hat{\rho} + p\frac{\hat{I}}{2}$$
$$= \left(1 - \frac{3p}{4}\right)\hat{\rho} + \frac{p}{4}(\hat{X}\hat{\rho}\hat{X} + \hat{Y}\hat{\rho}\hat{Y} + \hat{Z}\hat{\rho}\hat{Z})$$

best single-qubit input state

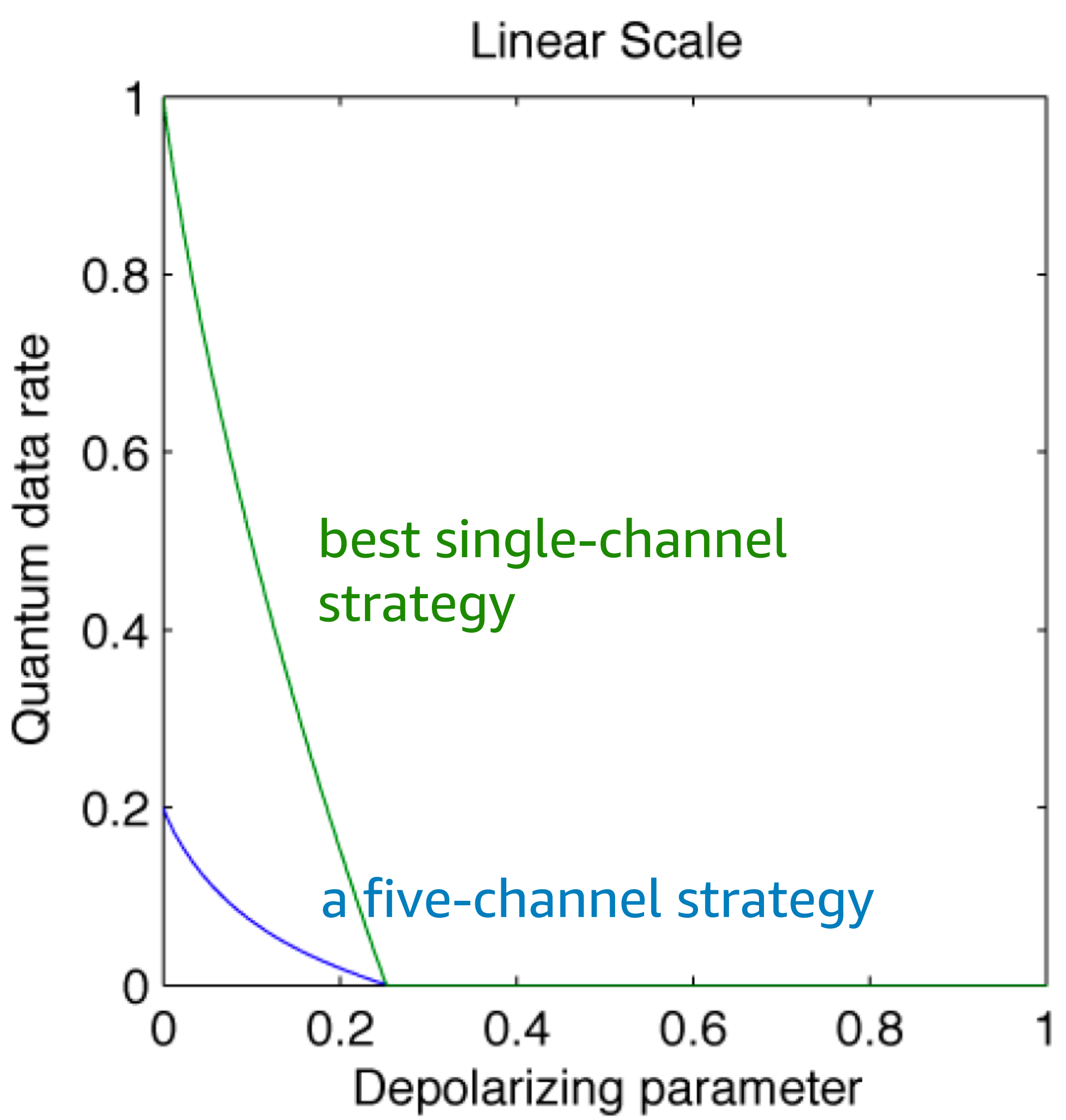
$$\hat{\rho}^\star = \arg\max_{\hat{\rho}} I_c(\mathcal{N}_D[p], \hat{\rho}) = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$

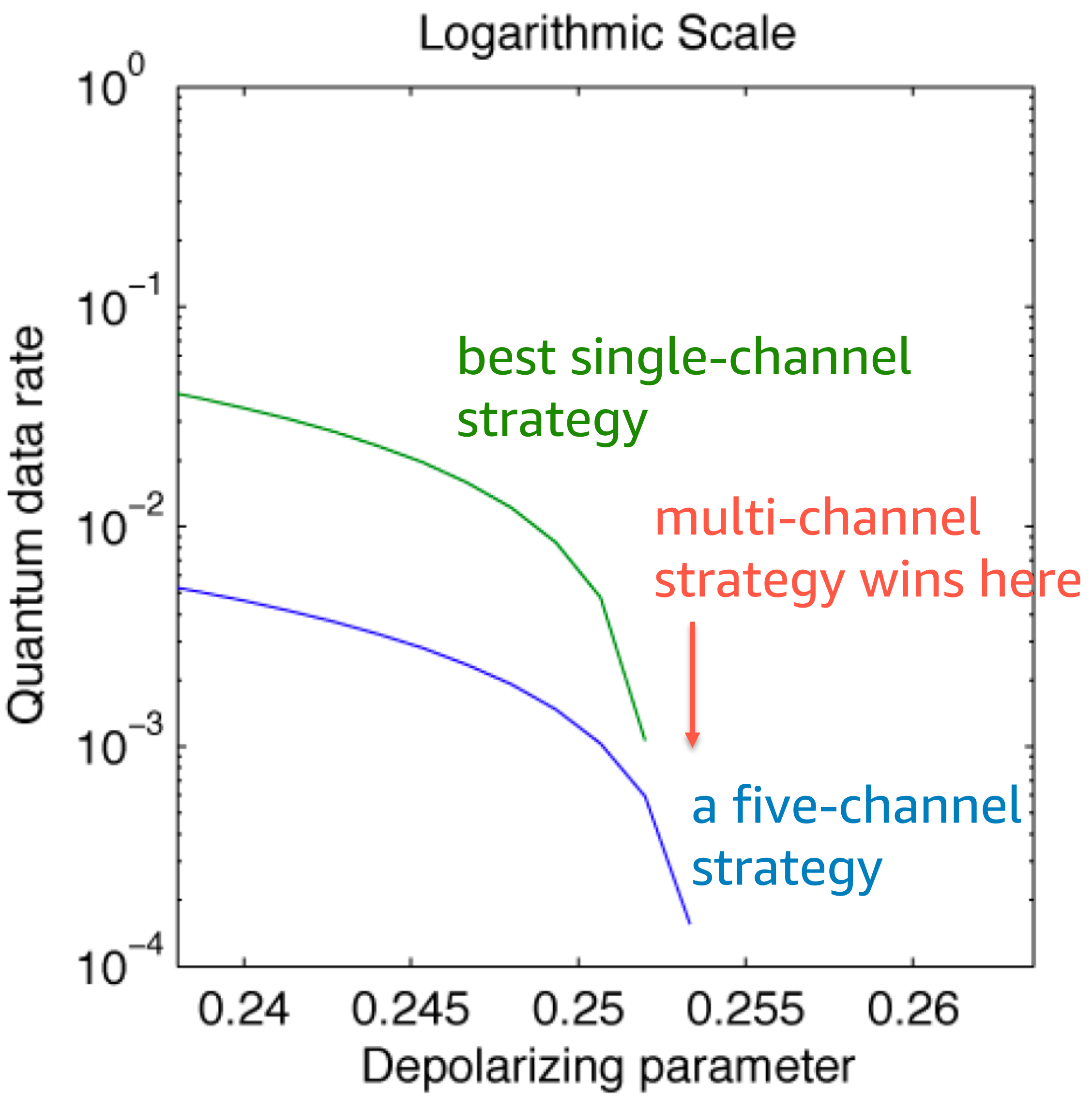
a five-qubit input state

$$\hat{\rho}^\star = \arg\max_{\hat{\rho}} I_c(\mathcal{N}_D[p]^{\otimes 5}, \hat{\rho}) = \frac{1}{2}|0\rangle\langle 0|^{\otimes 5} + \frac{1}{2}|1\rangle\langle 1|^{\otimes 5}$$

Qubit depolarization channel

figure from M. M. Wilde, arXiv:1106.1445 (see Fig. 24.4)





Qubit depolarization channel is superadditive!

Qubit depolarization channel: the way I saw it

$$\mathcal{N}_D[p](\hat{\rho}) = (1-p)\hat{\rho} + p\frac{\hat{I}}{2}$$
$$= \left(1 - \frac{3p}{4}\right)\hat{\rho} + \frac{p}{4}(\hat{X}\hat{\rho}\hat{X} + \hat{Y}\hat{\rho}\hat{Y} + \hat{Z}\hat{\rho}\hat{Z})$$

best single-qubit input state

$$\hat{\rho}^\star = \arg\max_{\hat{\rho}} I_c(\mathcal{N}_D[p], \hat{\rho}) = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$

an infinite-temperature single-qubit thermal state

a five-qubit input state

$$\hat{\rho}^\star = \arg\max_{\hat{\rho}} I_c(\mathcal{N}_D[p]^{\otimes 5}, \hat{\rho}) = \frac{1}{2}|0\rangle\langle 0|^{\otimes 5} + \frac{1}{2}|1\rangle\langle 1|^{\otimes 5}$$

a correlated multi-qubit thermal state

Taking the inspiration to bosonic systems

Taking the inspiration to bosonic systems

Single-mode thermal state

$$\hat{\tau}(\bar{n}) = \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} |n\rangle\langle n|$$

energy-constrained case ($\bar{n} < \infty$)

↓

for pure-loss channel (degradable):

	among single-mode states	including multi-mode states
among Gaussian states	optimal	optimal
including non-Gaussian states	optimal	optimal

for thermal-loss channel (not degradable):

	among single-mode states	including multi-mode states
among Gaussian states	optimal	shown to be not optimal
including non-Gaussian states	best-known	shown to be not optimal

will consider correlated multi-mode thermal states (Gaussian multi-mode states)

KN, S. Pirandola, L. Jiang, Nature Communications, **11**, 457 (2020)

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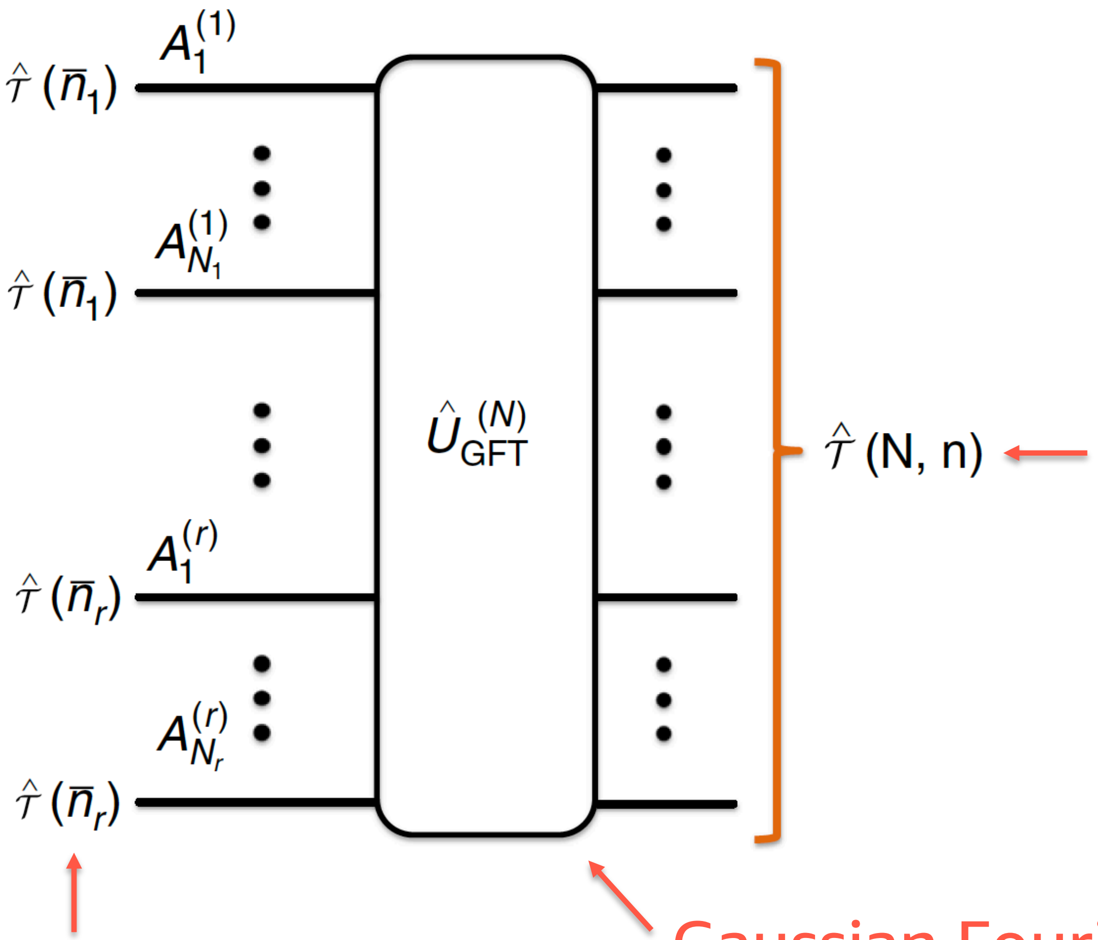
Taking the inspiration to bosonic systems

Correlated multi-mode thermal state

KN, S. Pirandola, L. Jiang, Nature Communications, **11**, 457 (2020)



correlated multi-mode thermal state: every mode has the same average photon number

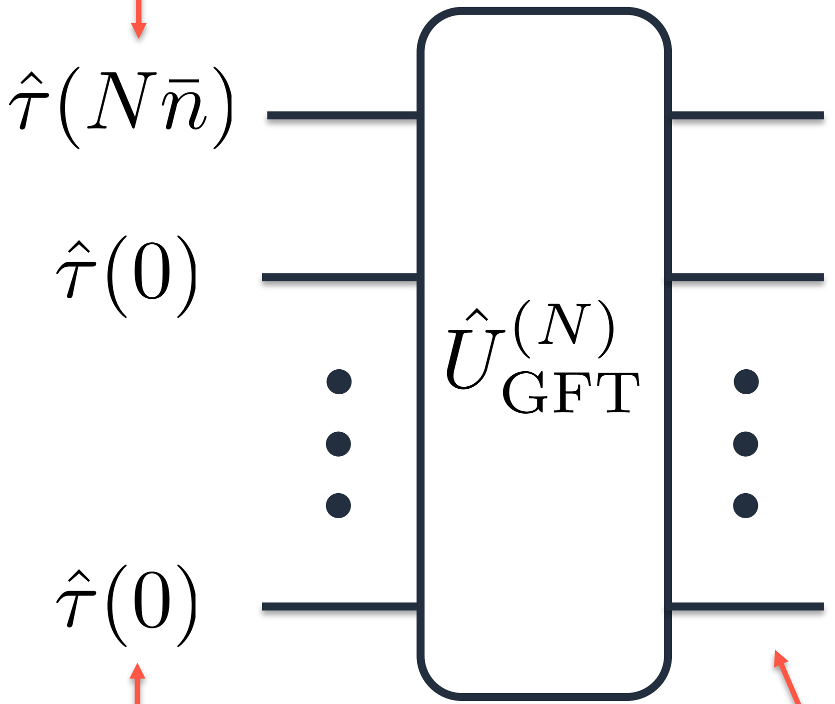
single mode thermal states: may have different average photon numbers

Gaussian Fourier transformation: mixing annihilation operators of the modes via discrete Fourier transformation

Taking the inspiration to bosonic systems

A simple example of correlated multi-mode thermal state

highly energetic thermal state



vacuum state

every mode has $\bar{n}$ photons on average

2x2 identity matrix: 2 because there are position and momentum operators in each mode

covariance matrix →

$$\begin{bmatrix} (\bar{n}+\frac{1}{2})I_2 & \bar{n}I_2 & \cdots & \bar{n}I_2 \\ \bar{n}I_2 & (\bar{n}+\frac{1}{2})I_2 & \cdots & \bar{n}I_2 \\ \vdots & \vdots & \ddots & \vdots \\ \bar{n}I_2 & \bar{n}I_2 & \cdots & (\bar{n}+\frac{1}{2})I_2 \end{bmatrix}$$

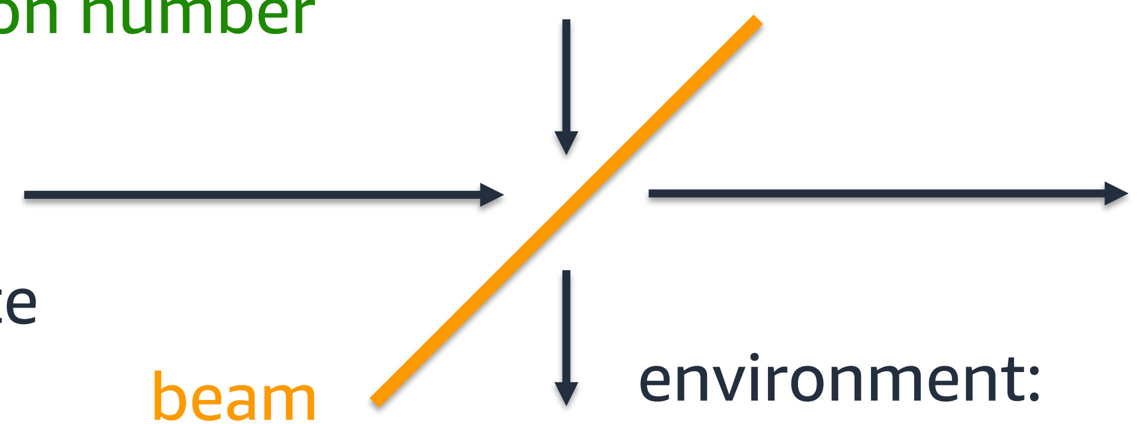
indicating every mode has $\bar{n}$ photons on average

non-trivial correlation between different modes

Main results

Problem formulation (setup)

Quantum communication over **lossy bosonic channels**



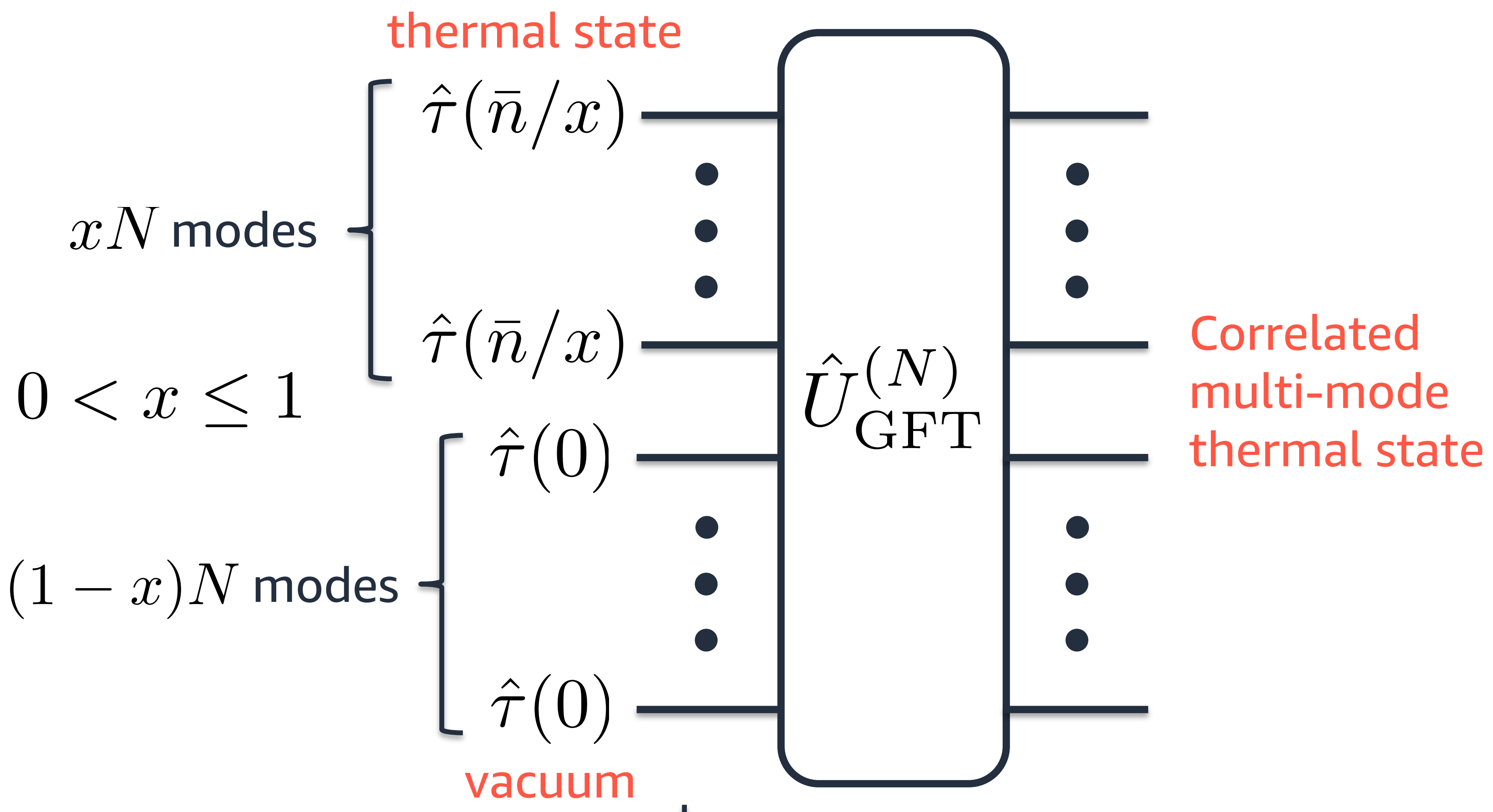
We are considering thermal-loss channels with $n_{th} > 0$ (not degradable) in the energy-constrained scenario $\bar{n} < \infty$

KN, S. Pirandola, L. Jiang, Nature Communications, **11**, 457 (2020)

Main results

KN, S. Pirandola, L. Jiang, Nature Communications, **11**, 457 (2020)

multi-channel strategy



$$xI_c\left(\mathcal{N}[\eta, n_{\mathrm{th}}], \hat{\tau}\left(\frac{\bar{n}}{x}\right)\right)$$

achievable rate (coherent information per channel use)

single-channel strategy

$$\hat{\tau}(\bar{n})$$

↓

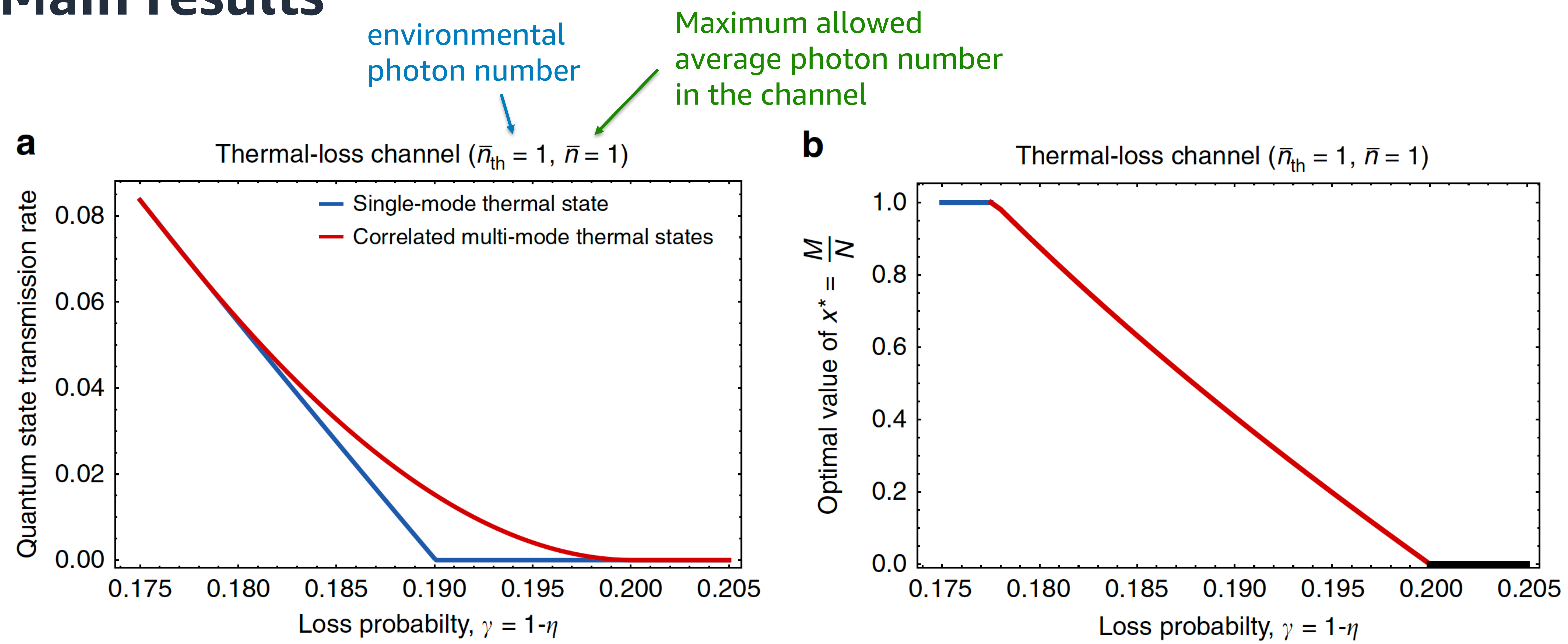
$$I_c(\mathcal{N}[\eta, n_{\mathrm{th}}], \hat{\tau}(\bar{n}))$$

achievable rate (coherent information)

vs.

Main results

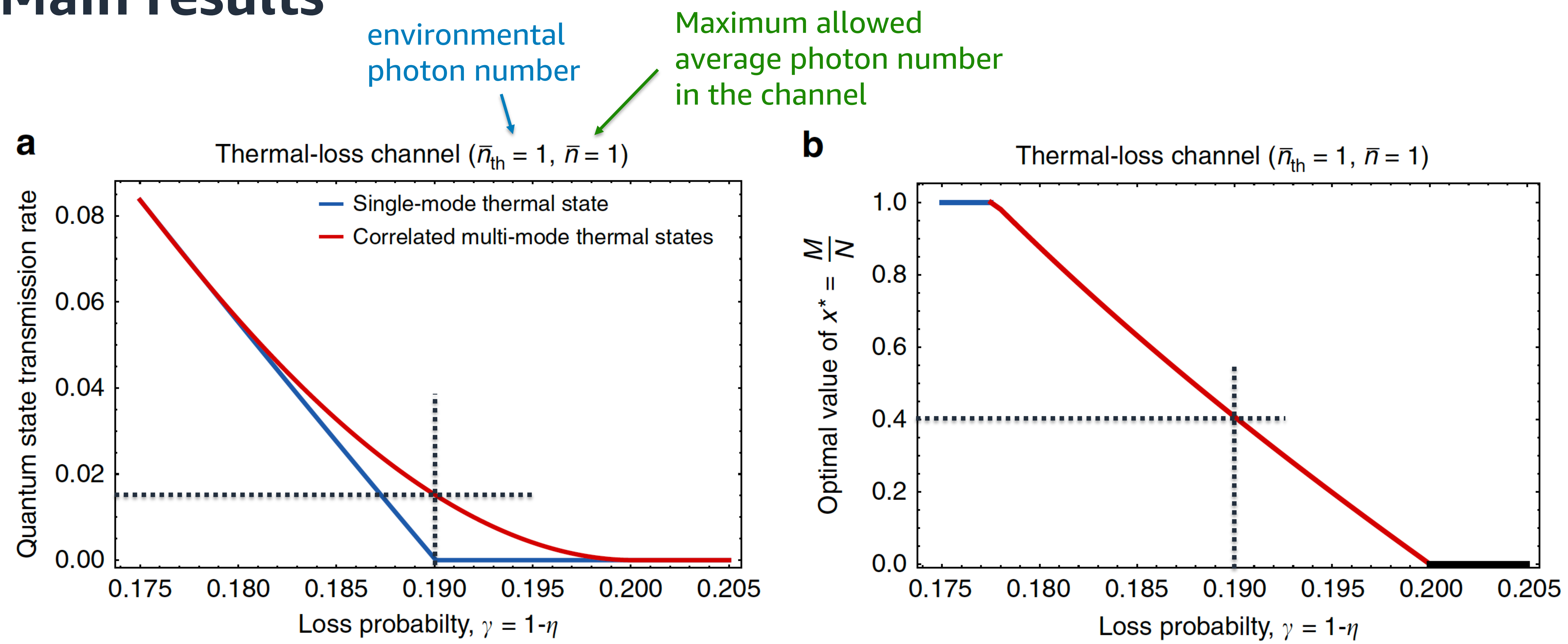
KN, S. Pirandola, L. Jiang, Nature Communications, **11**, 457 (2020)



Correlated multi-mode thermal states outperform single-mode thermal states in the high loss probability regime!

Main results

KN, S. Pirandola, L. Jiang, Nature Communications, **11**, 457 (2020)

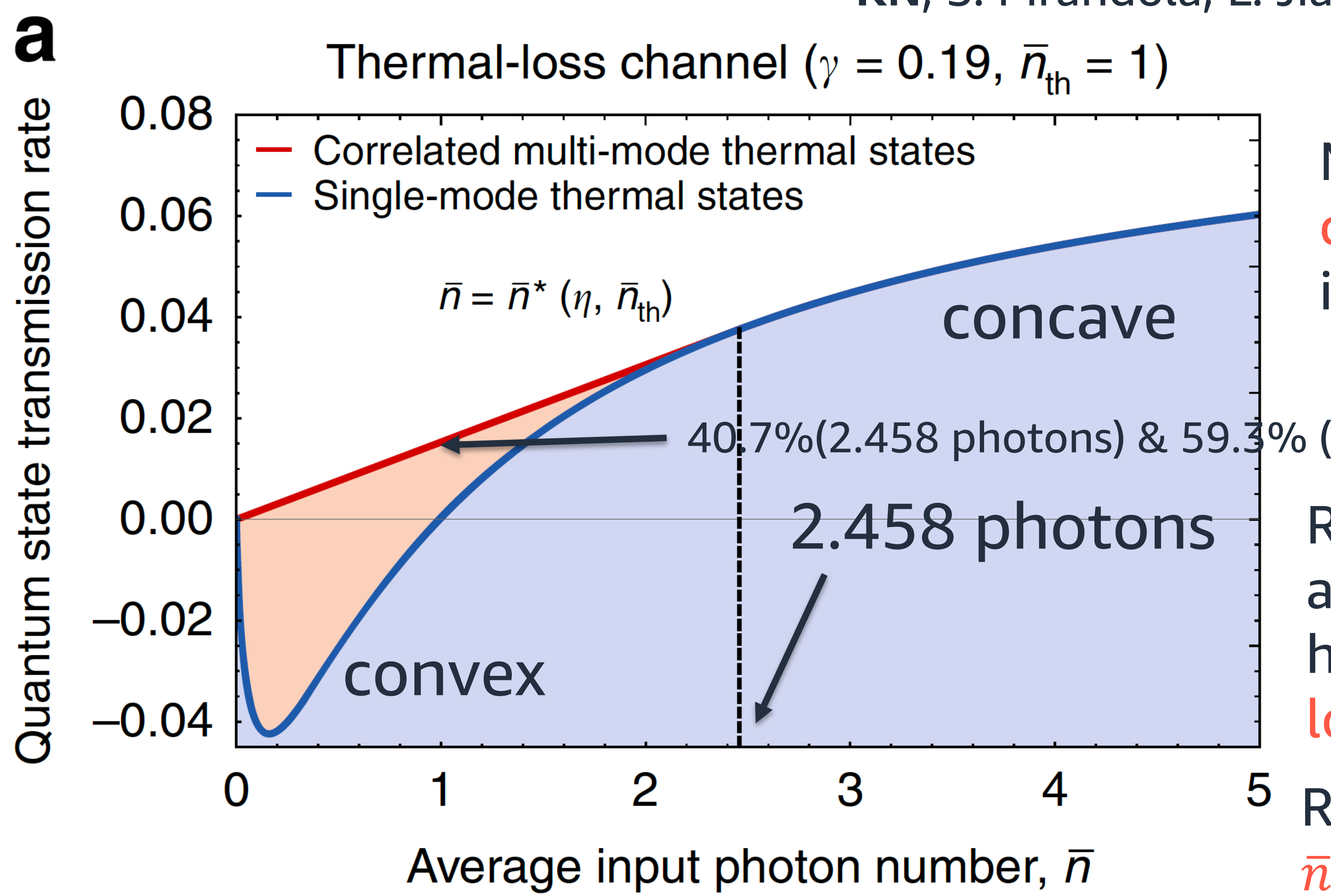


optimal strategy:

$x^\star = 0.407$ at $\gamma = 0.19$ ⟶ mixture of $\frac{\bar{n}}{x^\star} = 2.458$ (40.7%) and vacuum (59.3%) (mixed via Gaussian Fourier transformation)

Main results: convexity argument

KN, S. Pirandola, L. Jiang, Nature Communications, **11**, 457 (2020)



Mixing is advantageous due to the convexity of the coherent information in the low photon number regime!!

Remark 1: coherent information is always concave for degradable channels, hence no advantages like this for pure-loss channels

Remark 2: the advantage disappears for $\bar{n} \geq 2.458$ (for these specific parameters $\gamma = 0.19$ and $n_{th} = 1$)

optimal strategy:

$x^\star = 0.407$ at $\gamma = 0.19$ ⟶ mixture of $\frac{\bar{n}}{x^\star} = 2.458$ (40.7%) and vacuum (59.3%)
(mixed via Gaussian Fourier transformation)

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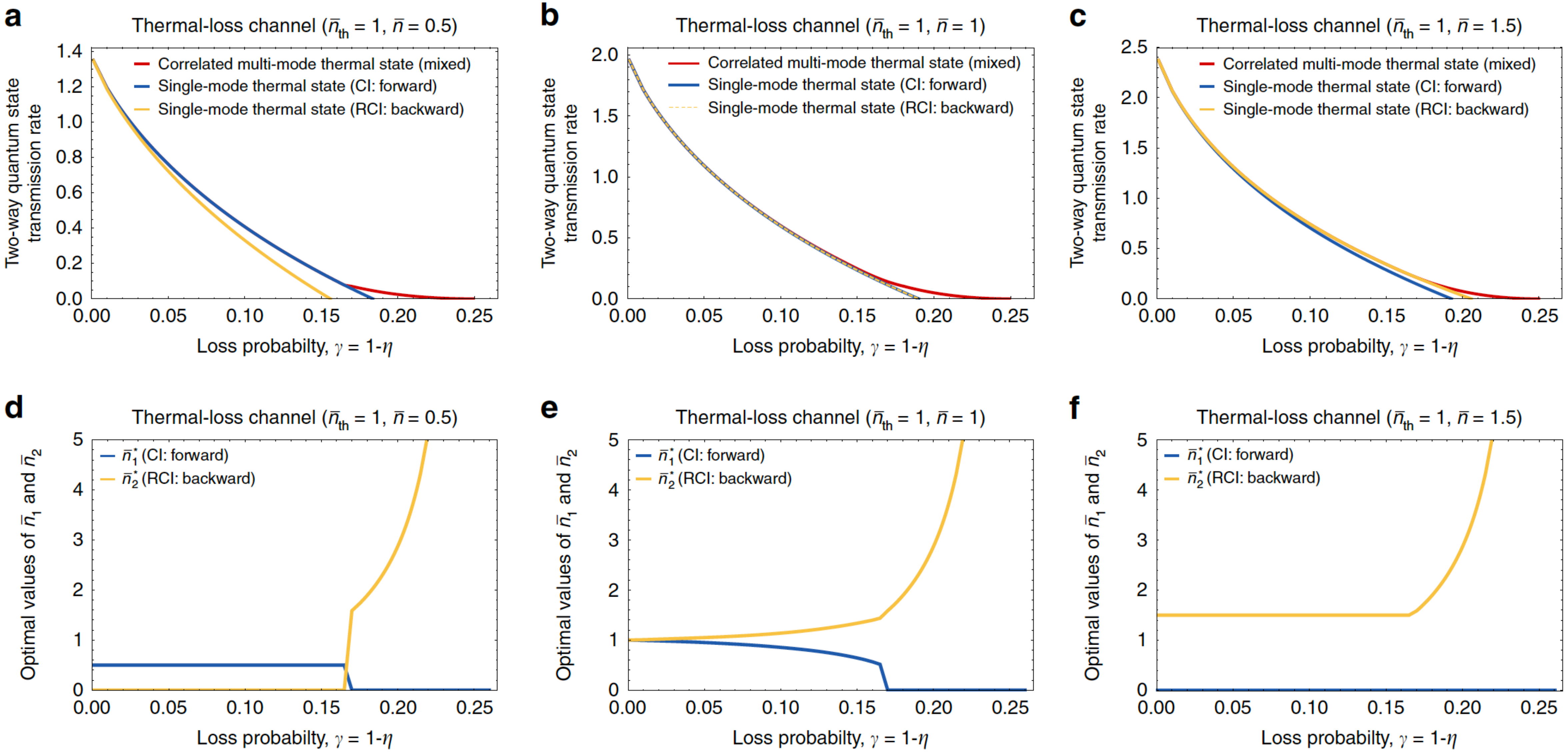
Main results: two-way communication

KN, S. Pirandola, L. Jiang, Nature Communications, **11**, 457 (2020)



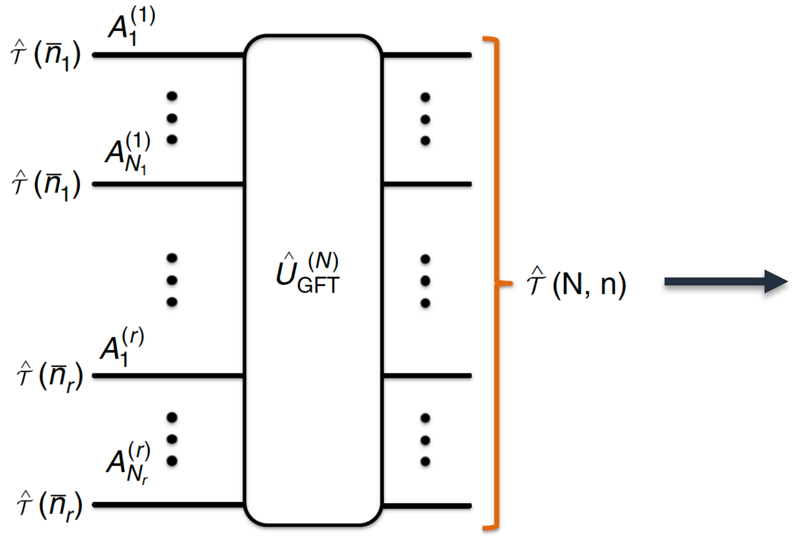
Mixing and matching forward and backward strategies turns out to be advantageous in the two-way communication scenario!

Outlook

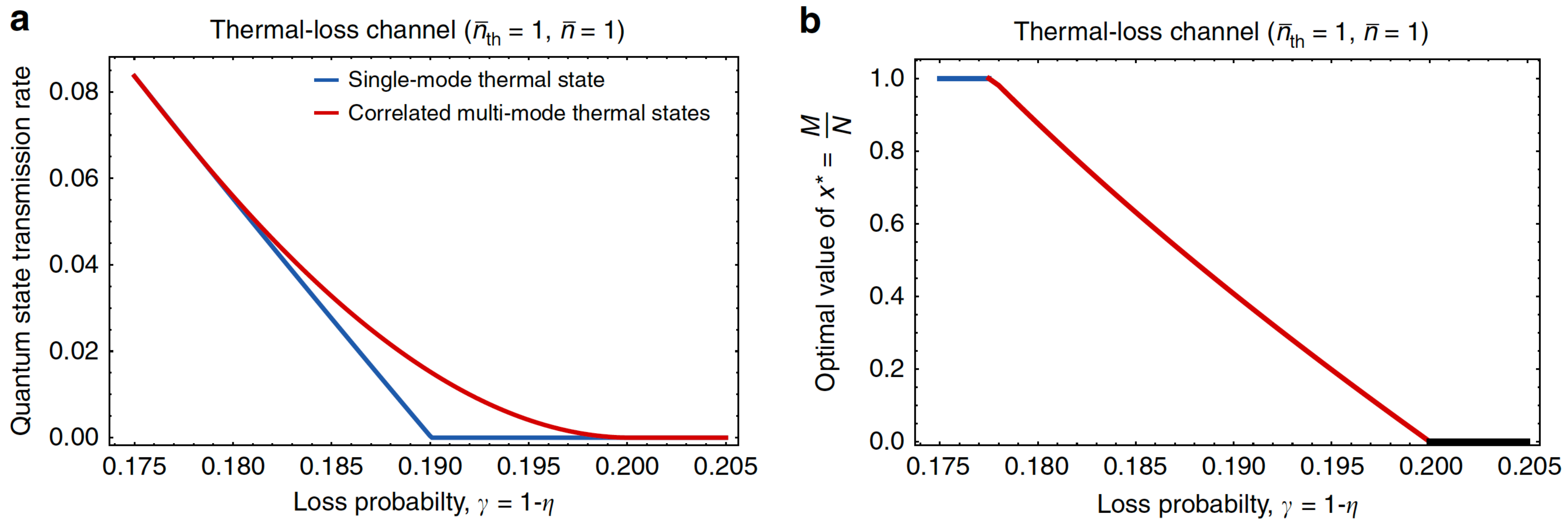
Outlook

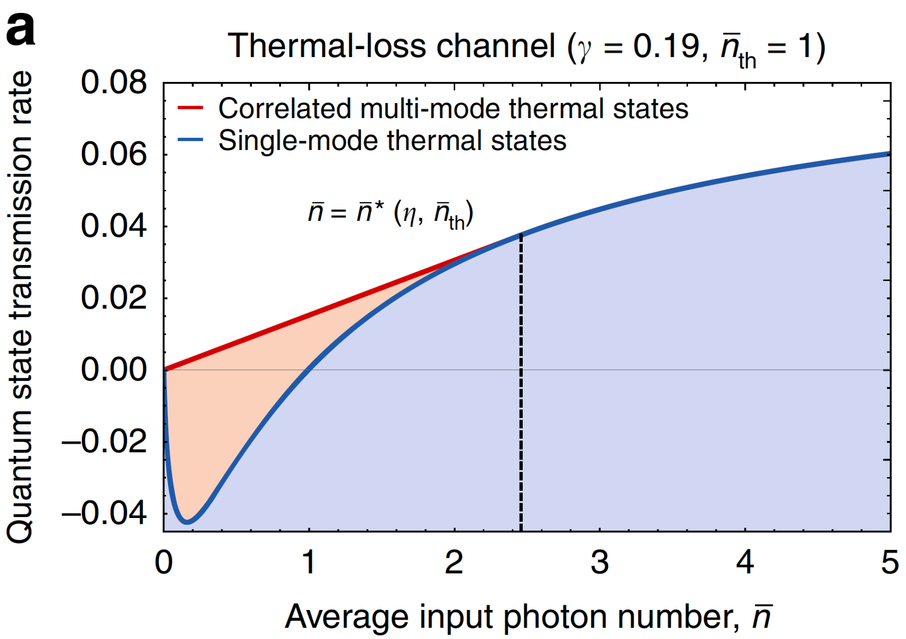
KN, S. Pirandola, L. Jiang, Nature Communications, **11**, 457 (2020)

Correlated multi-mode thermal state



Tightest lower bound of the thermal-loss channel capacity (via correlated multi-mode thermal state)





+two-way communication
+private communication
(check out the paper for more details)

Convexity argument

applicable to other channels as well??

any advantages in using non-Gaussian input states?

In collaboration with

Stefano Pirandola

Liang Jiang

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