Emergent classicality and a bound on the spread of quantum information

Joint work with Xiao-Liang Qi

Speaker: Daniel Ranard

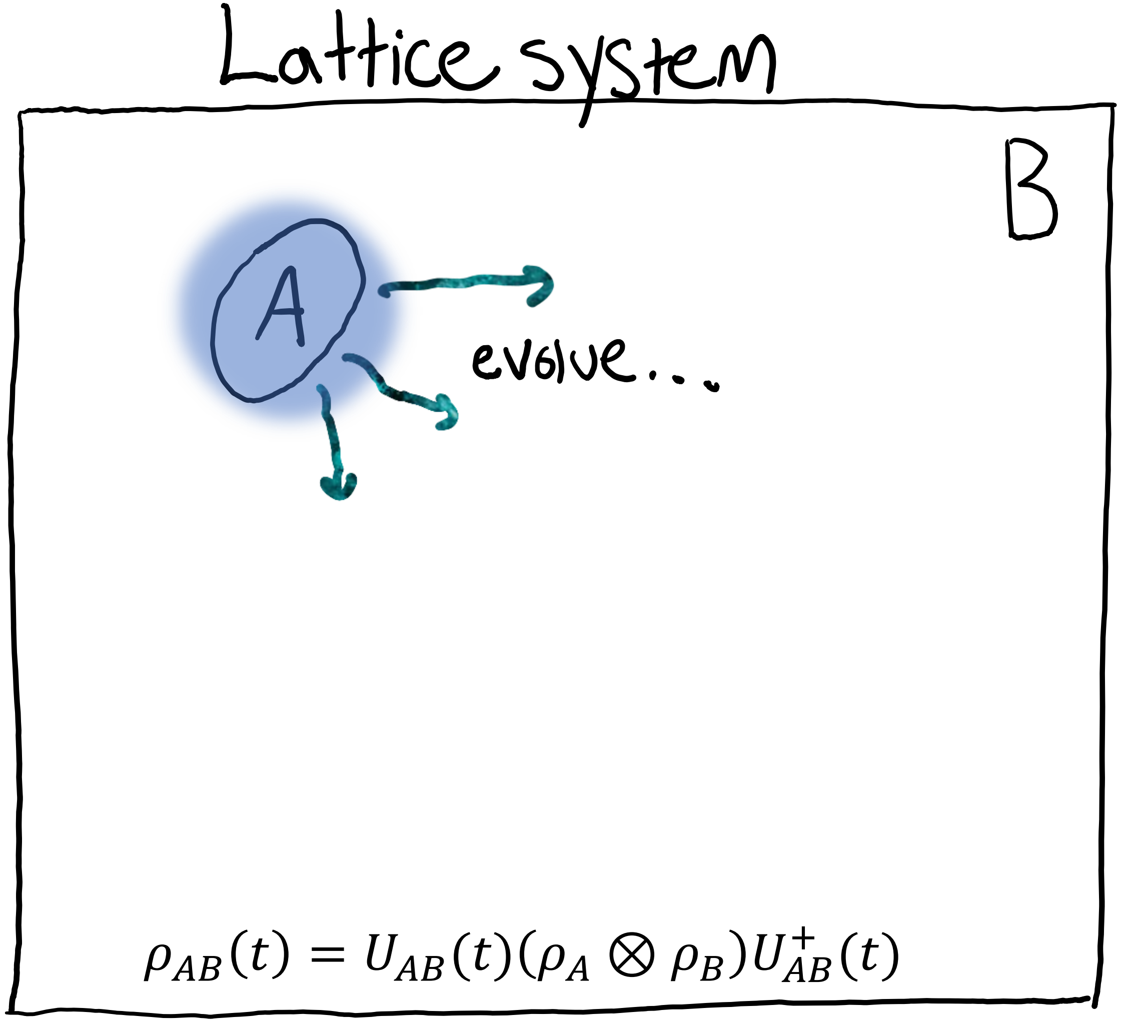
arXiv: 2001.01507

Builds on: Brandão, Piani, Horodecki (2015*, Nat. comm. 6:7908)*

Outline

- Information spreading in different systems
- General constraint on spreading in *all* systems
- Theorem statement
- Emergent classicality in many-body systems

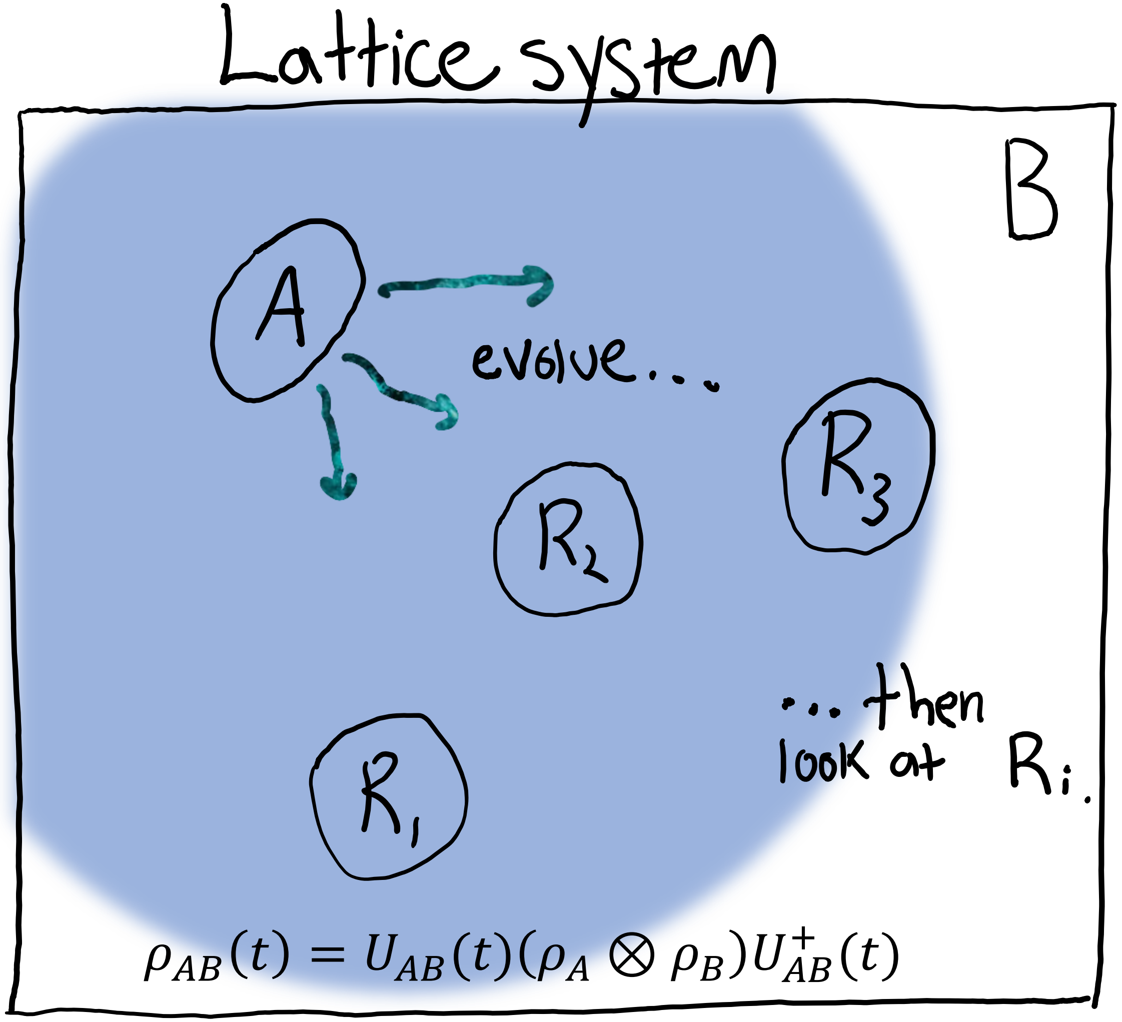
How can information spread during evolution?



$$\rho_{AB}(t) = U_{AB}(t)(\rho_A \otimes \rho_B)U_{AB}^{+}(t)$$

Information spreads within lightcone.

How can information spread during evolution?



$\rho_{AB}(t) = U_{AB}(t)(\rho_A \otimes \rho_B)U_{AB}^{+}(t)$

Information spreads within lightcone.

What can we learn about A, just looking at local region R_i?

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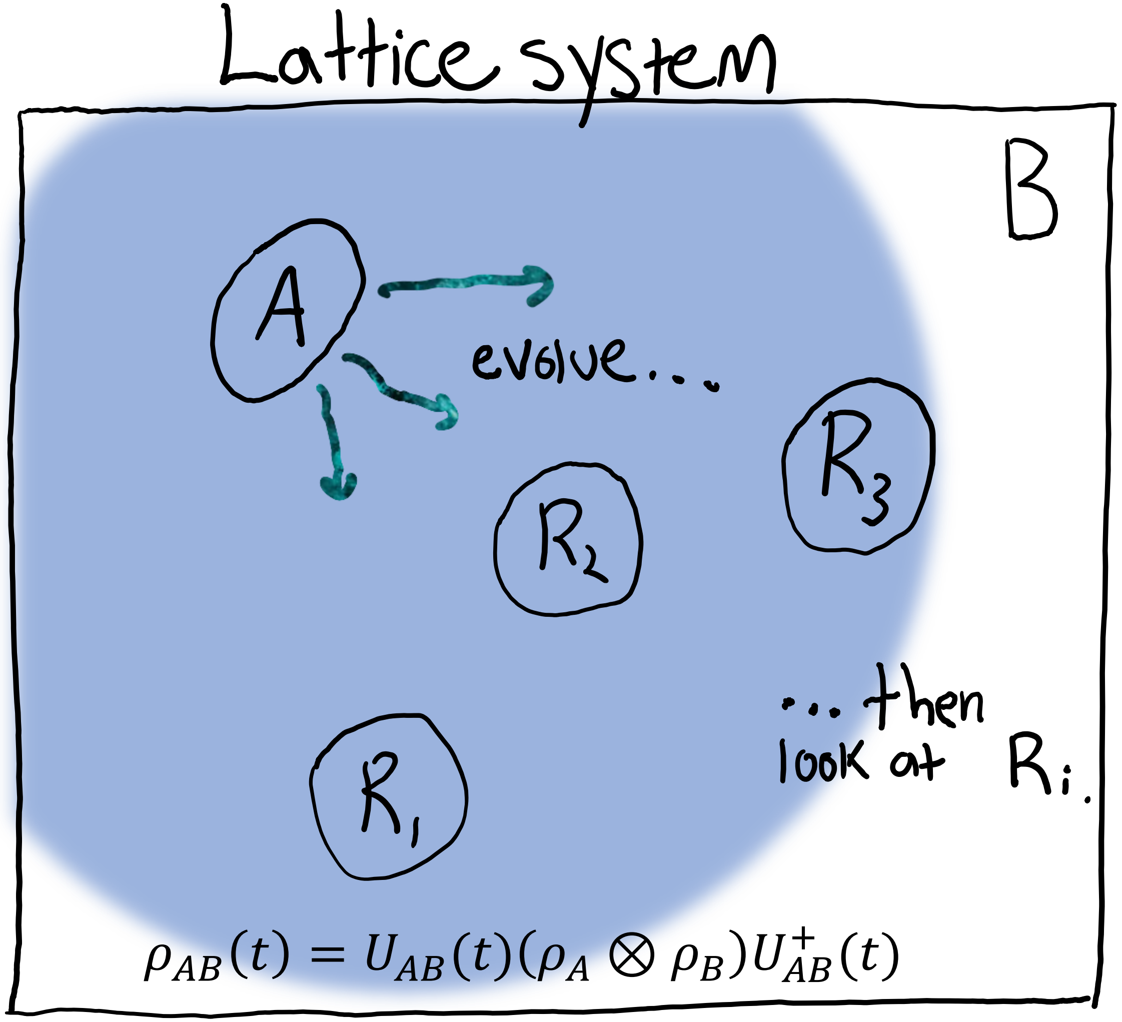

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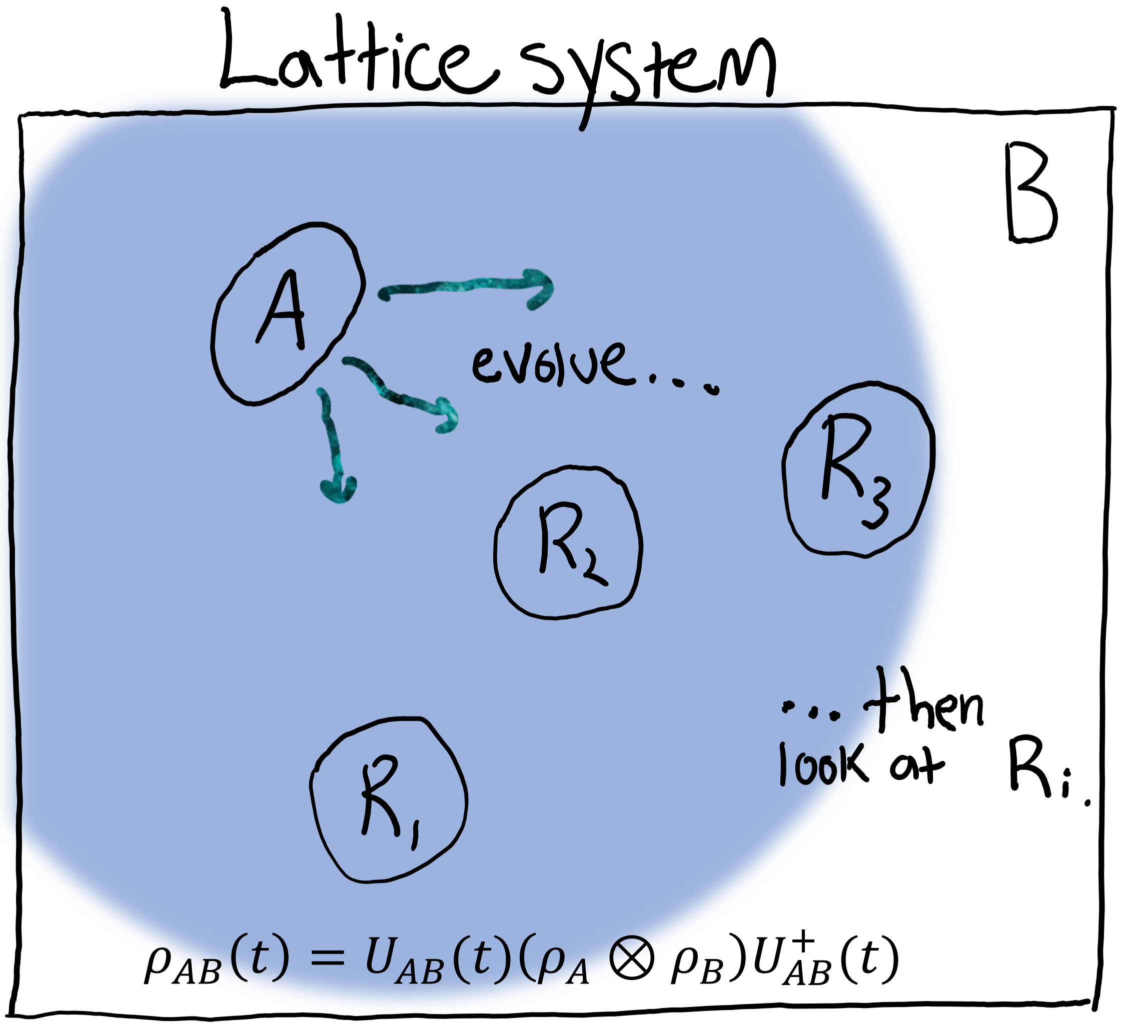
What can we learn about A, just looking at local region R_i?

Example situations:

- Long random circuit, or thermalization
- Direct transport $A \to R_1$ and ~~$A \to R_2$~~
- Decoherence: “Measurement” of A by environment B; results passed to each R_i

How can information spread during evolution?



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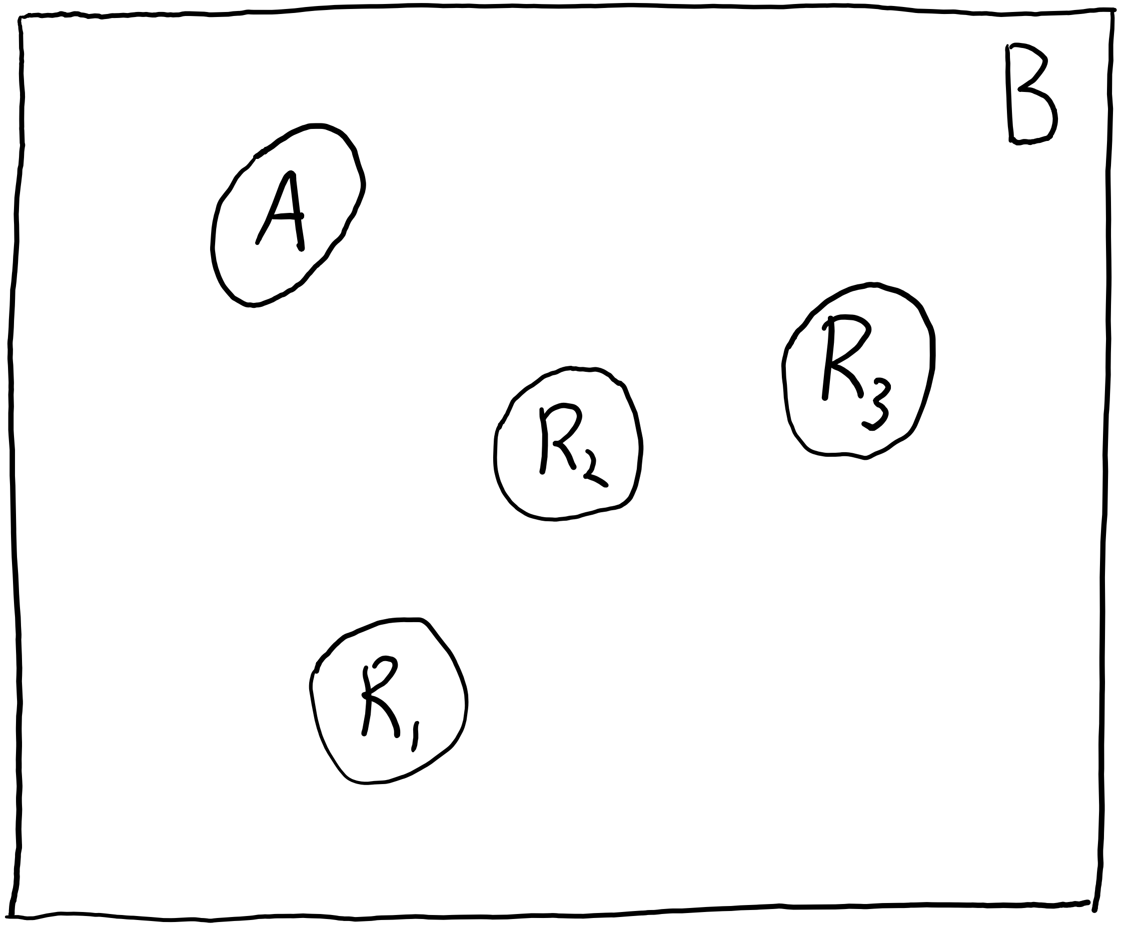
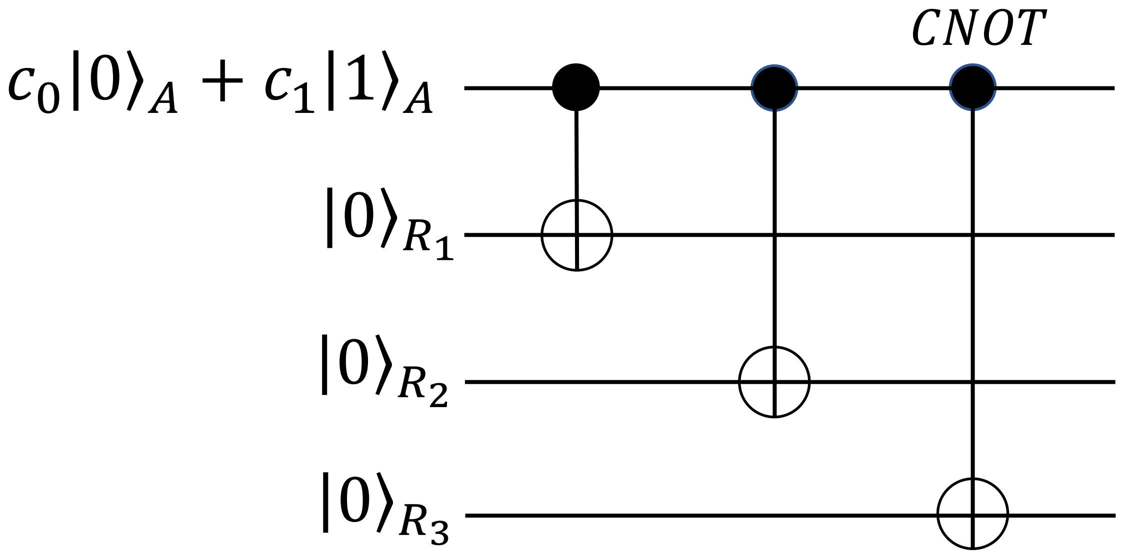
Decoherence example





$c_0|0\rangle_A + c_1|1\rangle_A$

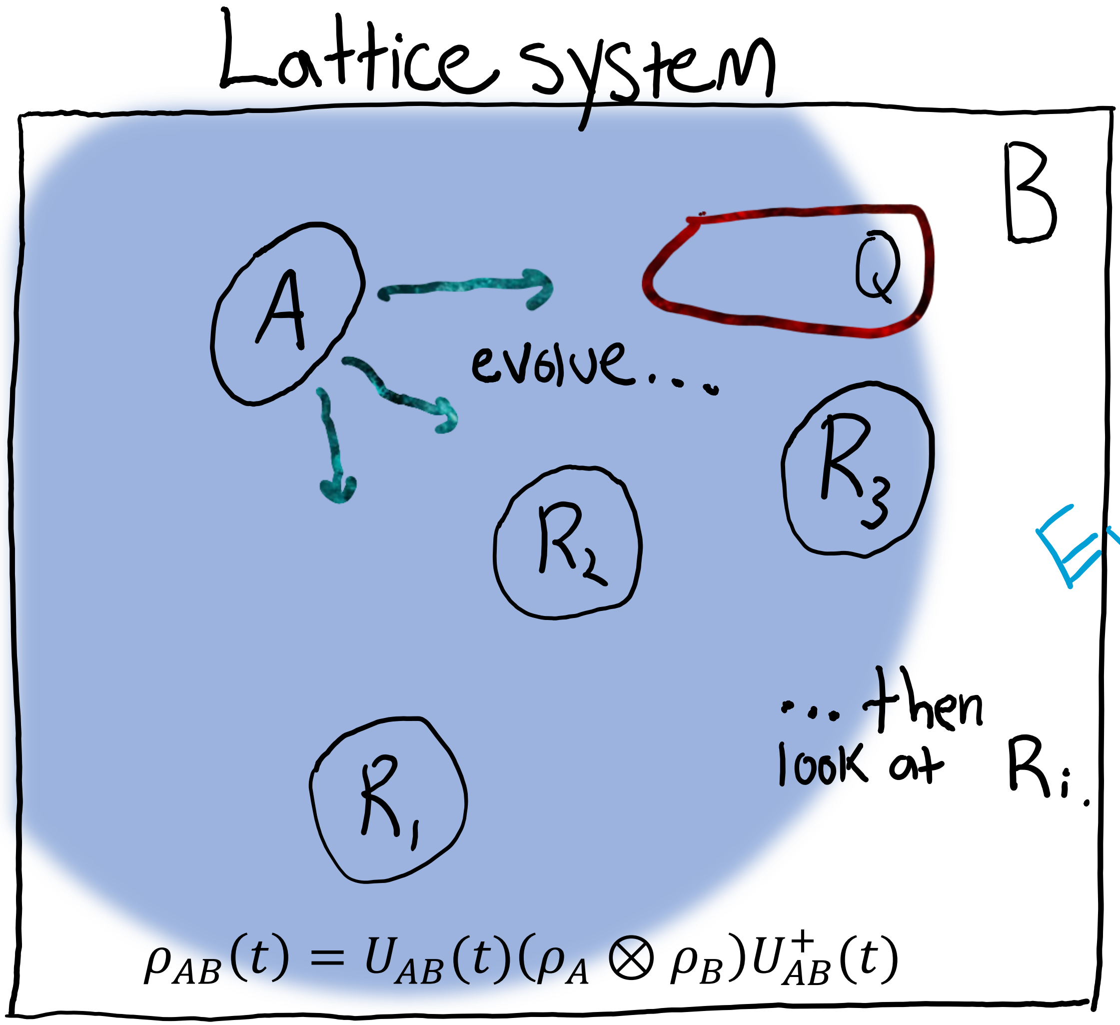
$|0\rangle_{R_1}$

$|0\rangle_{R_2}$

$|0\rangle_{R_3}$

Output: $\rho_{R_i} = |c_0|^2|0\rangle\langle 0| + |c_1|^2|1\rangle\langle 1|$

How can information spread during evolution?



$$\rho_{AB}(t) = U_{AB}(t)(\rho_A \otimes \rho_B)U_{AB}^{+}(t)$$

What can we learn about A, just looking at local region R_i?

Example situations:

Everything!

- Long random circuit, or thermalization
- Direct transport $A \to R_1$ and ~~$A \to R_2$~~
- Decoherence: “Measurement” of A by environment B; results passed to each R_i

Earlier work

Builds on prior work introducing this style of bound:

Generic emergence of classical features in quantum Darwinism, Brandão, Piani, Horodecki.

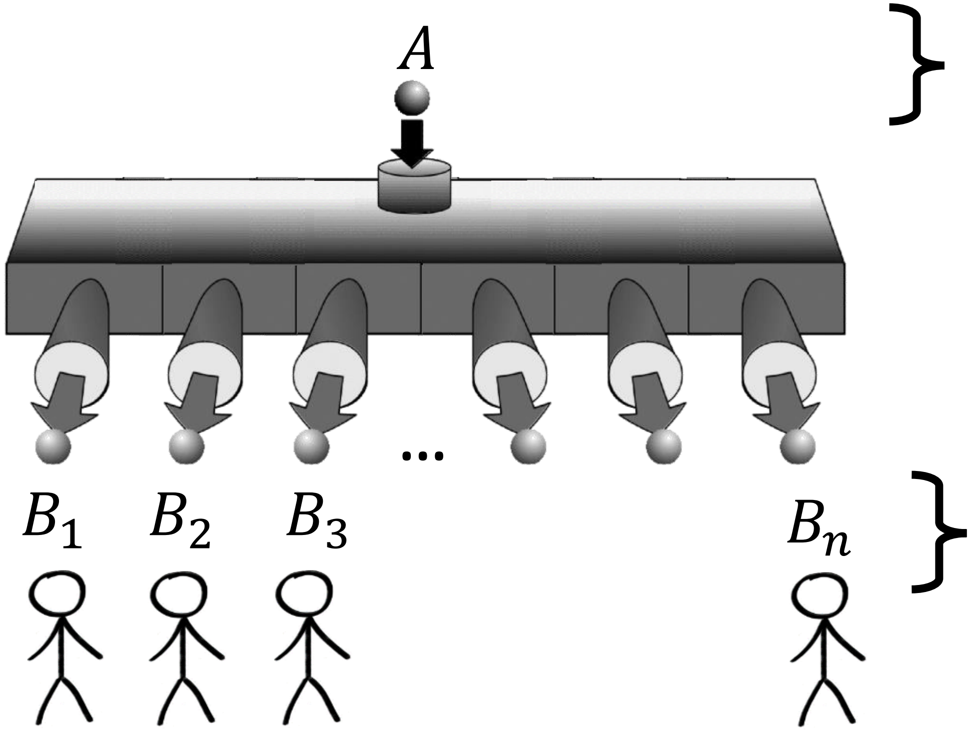
The present work proves much stronger monogamy-style statement, with a simple + constructive argument, allowing a numerical algorithm.

History of related techniques:

arXiv:1010.1750 (Brandão, Christandl, Yard)

arXiv:1210.6367 (Brandão, Harrow)

Studying local sub-systems of the environment

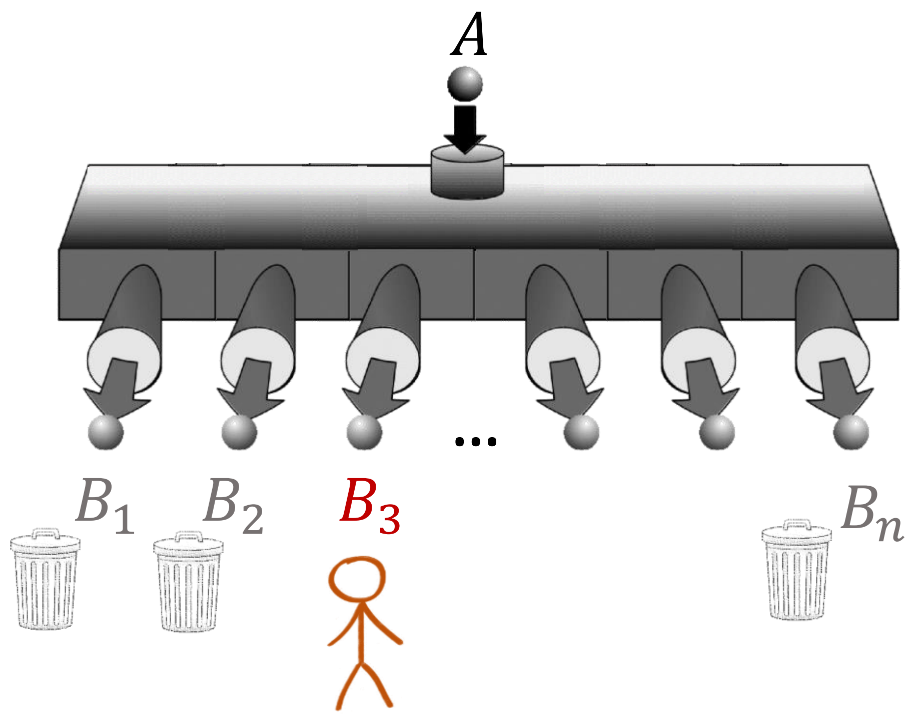


Input system

Imagine input $\boldsymbol{A}$ couples to environment $\boldsymbol{B}$, then study local sub-systems of $\boldsymbol{B}$.

$B = B_1 \otimes \cdots \otimes B_n$ Output, or “environment”

Studying local sub-systems of the environment



Input system

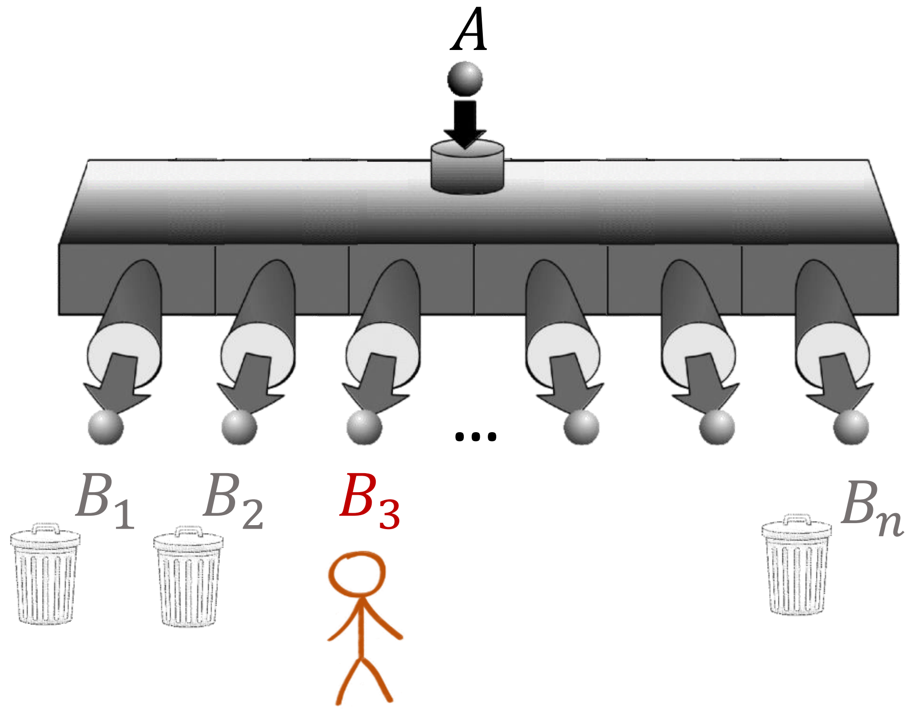
Imagine input A couples to environment B, then study local sub-systems of B.

$B = B_1 \otimes \cdots \otimes B_n$ Output, or “environment”

What can Bob learn about A? For most B_i, only classical information! (Our result)

Almost everywhere in the environment B, the locally accessible information about A looks *classical*, i.e. can be obtained from a measurement of A in some fixed basis.

Studying local sub-systems of the environment



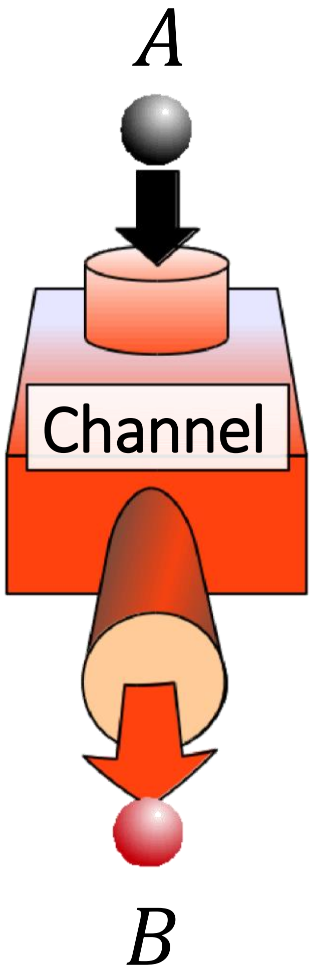
ρ_A

Model evolution as coupling to environment, evolving, then tracing out all except B_i

$$\rho_{B_i} = Tr_{A\,\overline{B_i}}\,(U_{AB}(\rho_A \otimes \tau_B)U_{AB}^{+})$$

Environment starts in state τ_B
Then both systems evolve by unitary U_{AB}

Interlude: Measure-and-prepare channels



Quantum channels $A \to B$ are maps from the space of density operators on system A to density operators on B, i.e.

$\rho_A \mapsto \rho_B$

Entanglement breaking

"Measure-and-prepare" channel: Special type of channel that takes the form

$$\rho_A \mapsto \rho_B = \sum_{\alpha} \mathrm{Tr}(M^{\alpha} \rho_A)\, \sigma_B^{\alpha}$$

for some measurement operators $\{M^{\alpha}\}_{\alpha}$ and states $\{\sigma_B^{\alpha}\}_{\alpha}$ (e.g. orthogonal projectors $M^{\alpha} = |\alpha\rangle\langle\alpha|$)

Interlude: Measure-and-prepare channels

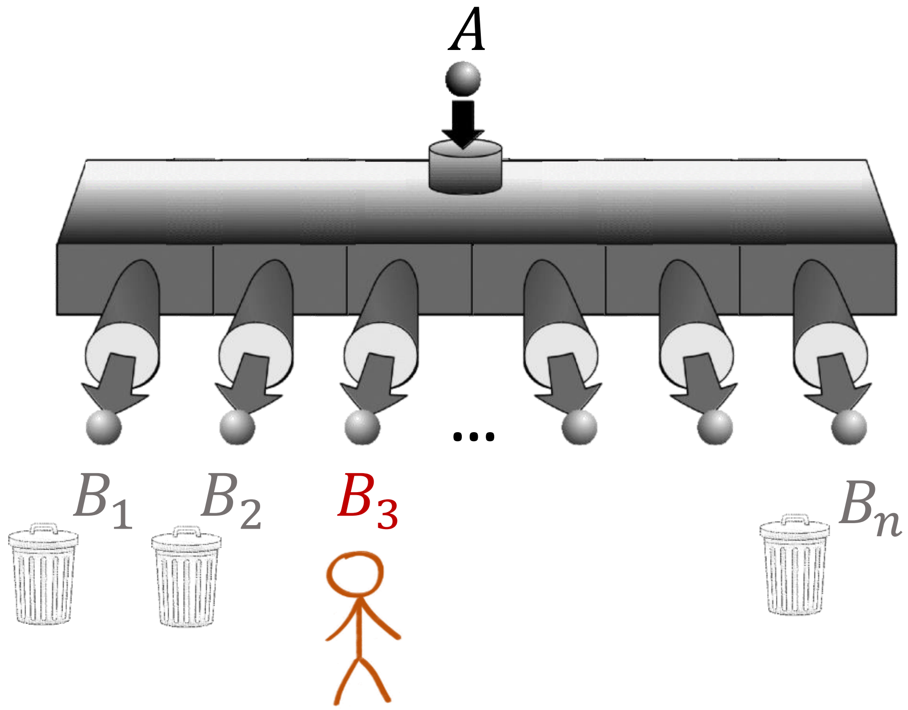
Evolutions of the form

$$\rho_A \mapsto \rho_B = \sum_\alpha \mathrm{Tr}(M^\alpha \rho_A)\, \sigma_B^\alpha \,,$$

represent measuring A in the basis associated to M^α and then preparing the state σ_B^α contingent on classical outcome α.

For Alice and Bob at different labs A and B, they can implement such a map by sending only classical information.

Our result: For any evolution $A \to B$ …



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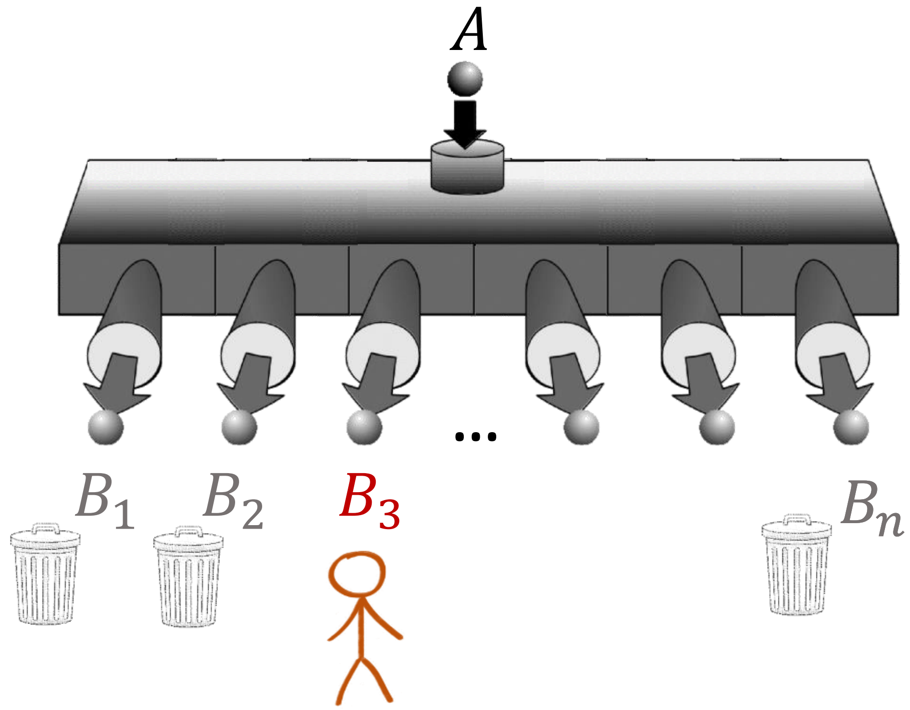


ρ_A

Model evolution as coupling to environment, evolving, then tracing out all except B_i

$$\rho_{B_i} = Tr_{A\,\overline{B_i}}\left(U_{AB}(\rho_A \otimes \tau_B)U_{AB}^{+}\right)$$

$$\boxed{\approx \sum_{\alpha} \mathrm{Tr}(M^{\alpha}\rho_A)\,\sigma_{B_i}^{\alpha} \quad \text{“measure-and-prepare”}}$$

for almost all B_i

for some choice of measurement operators $\{M^{\alpha}\}$

↳ independent of B_i

For almost all B_i (all but O(1)-many), the evolution $\rho_A \to \rho_{B_i}$ looks like performing a fixed classical measurement on A, followed by preparing some state on B_i based on the outcome.

Examples of applying theorem to evolutions

ρ_A

Model evolution as coupling input A to environment B,
evolving both, then tracing out all except B_i

$$\rho_{B_i} = \mathrm{Tr}_{A\,\overline{B_i}}\left(U_{AB}(\rho_A \otimes \tau_B)U_{AB}^{+}\right)$$

Example: Direct transport $A \to B_1$

ρ_A

Model evolution as coupling input A to environment B, evolving both, then tracing out all except B_i

$$\rho_{B_i} = \mathrm{Tr}_{A\,\overline{B_i}}\left(U_{AB}(\rho_A \otimes \tau_B)U_{AB}^{+}\right)$$

$$\tau_B = |0\rangle^{\otimes n}\langle 0|^{\otimes n}$$

$$U_{AB}|0\rangle_A|0\ldots 0\rangle_B = |0\rangle_A|0\ldots 0\rangle_B$$

$$U_{AB}|1\rangle_A|0\ldots 0\rangle_B = |0\rangle_A|10\ldots 0\rangle_B$$

Example:
Input system A sent faithfully to B_1
Other B_i sent to $|0\rangle$ (for $i > 1$)

$\rho_A \to \rho_{B_1} = \rho_A$

Not measure-and-prepare ✗

$\rho_A \to \rho_{B_i} = |0\rangle\langle 0|_{B_i}$ (for $i > 1$)

Measure-and-prepare ✓ (Trivial prep.)

Example: Decoherence

ρ_A

$\rightsquigarrow$ Model evolution as coupling input A to environment B, evolving both, then tracing out all except B_i

$$\rho_{B_i} = \mathrm{Tr}_{A\,\overline{B_i}}\left(U_{AB}(\rho_A \otimes \tau_B)U_{AB}^{+}\right)$$

$$\tau_B = |0\rangle^{\otimes n}\langle 0|^{\otimes n}$$

$$U_{AB}|0\rangle_A|0\rangle_B{}^{\otimes n} = |0\rangle_A|0\rangle_B{}^{\otimes n}$$

$$U_{AB}|1\rangle_A|0\rangle_B{}^{\otimes n} = |1\rangle_A|1\rangle_B{}^{\otimes n}$$

Example:
Input system A is measured/decohered in $|0\rangle, |1\rangle$ basis
Outcome recorded on each B_i

$$\rho_A \to \rho_{B_i} = Tr(\,|0\rangle\langle 0|\,\rho_A)|0\rangle\langle 0|_{B_i} + Tr(\,|1\rangle\langle 1|\,\rho_A)|1\rangle\langle 1|_{B_i}$$

Measure-and-prepare ✓

Example: Spin chain evolution

ρ_A

$\downarrow$ Model evolution as coupling input A to environment B, evolving both, then tracing out all except B_i

$$\rho_{B_i} = \mathrm{Tr}_{A\,\overline{B_i}}\,(U_{AB}(\rho_A \otimes \tau_B)U_{AB}^{+})$$

τ_B = Groundstate of spin chain B

U_{AB} = Evolution of extended chain AB

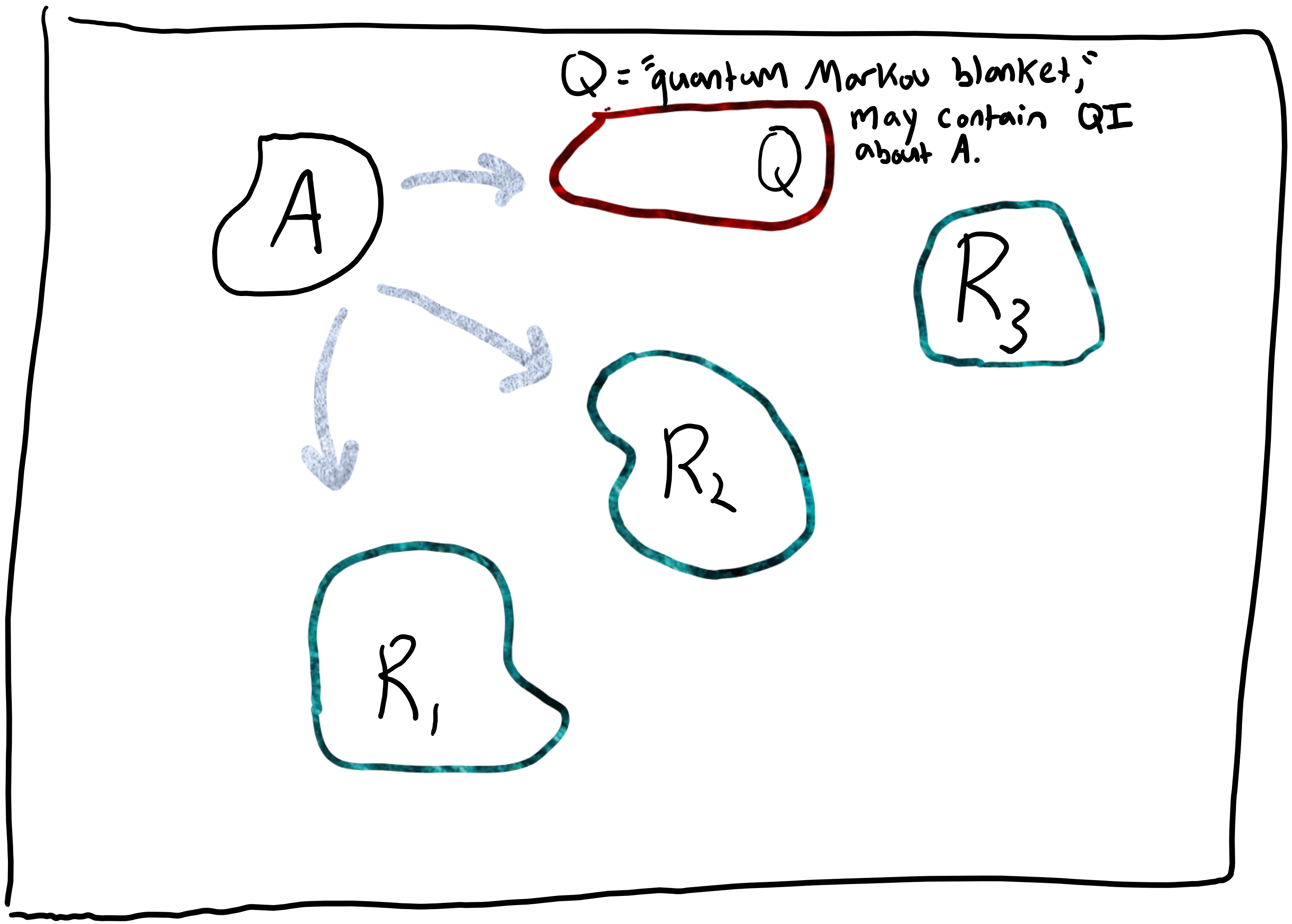
Example:
Couple qubit A onto end of spin chain B, then evolve extended chain

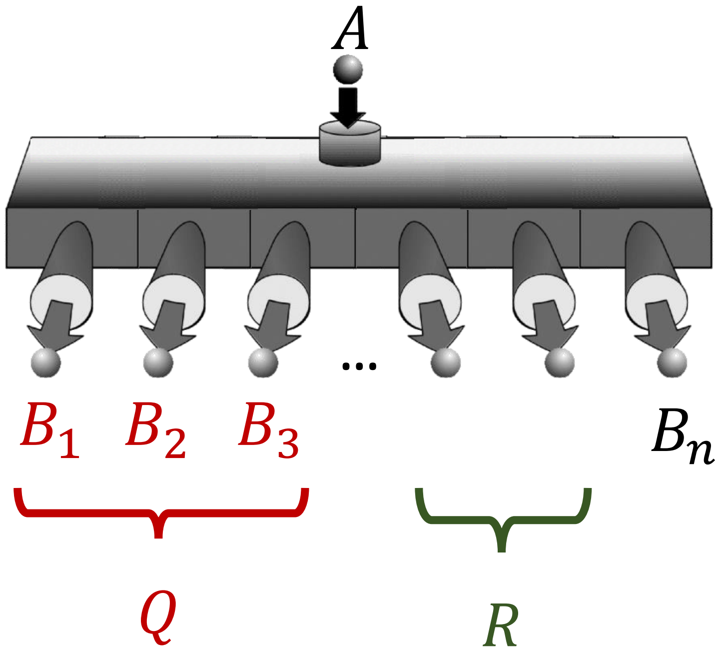
$\rho_A \to \rho_{B_i} =$????

Numerical examples work!

Measure-and-prepare ✓

Theorem statement (almost)





Excluded region: "Quantum Markov blanket"

On all regions R away from Q, evolution $A \rightarrow R$ is approx. measure-and-prepare.

$|Q| = O(1)$, doesn't scale with total system size!

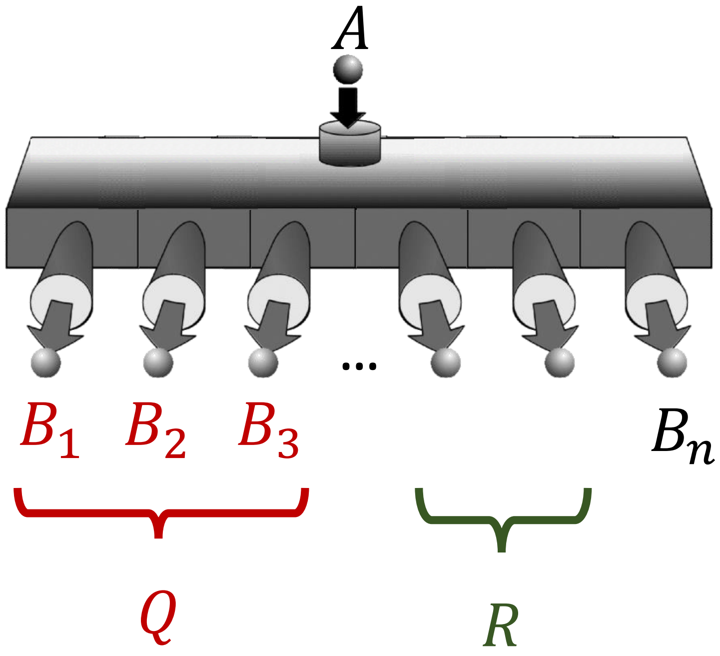
Quantum Markov blanket, Q

Q roughly includes outputs B_i with most information about A

Q "blankets" A:
Any information about A that's accessible on small regions R outside Q can be obtained from a classical measurement on just Q.

Q includes, at least, any region with locally accessible *quantum* information about A.

For arbitrarily large environments, you can still "cover" A with an O(1)-sized blanket Q !



Excluded region:
"Quantum Markov blanket"

On all regions R away from Q, evolution $A \rightarrow R$ is approx. measure-and-prepare.

$|Q| = O(1)$, doesn't scale with total system size!

Theorem statement

Consider arbitrary quantum channel

$$N : \mathcal{D}(A) \to \mathcal{D}(B_1 \otimes \cdots \otimes B_n)$$

$\mathcal{D}(X)$ = density operators on X

Theorem: For any $\epsilon > 0$ and $r \in \{1, \dots, n\}$, there exists a POVM $\{M_\alpha\}$ and an "excluded" output subset $Q \subset \{B_1, \dots, B_n\}$ of size

$$|Q| \leq \frac{1}{\epsilon^2} 2 r d_A^2 \ln d_A$$

$d_A = \dim(A)$

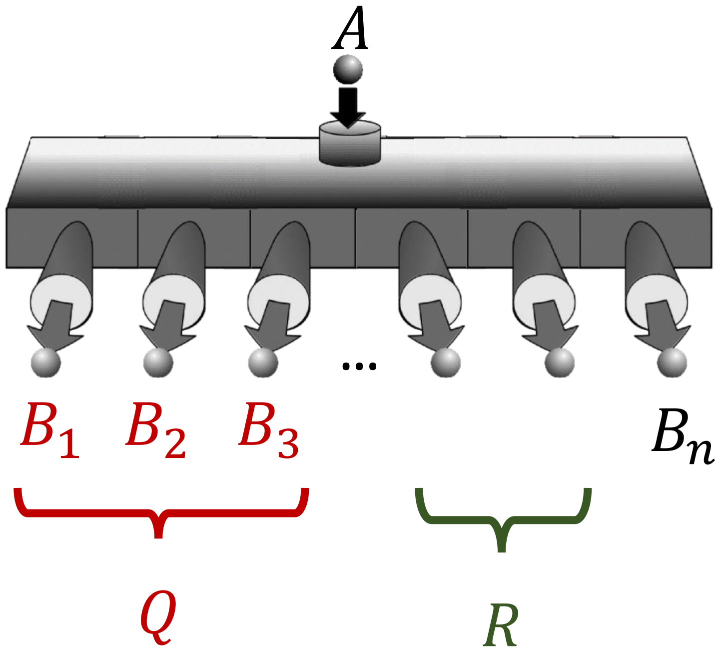
such that:
For all output subsets $R \subset \{B_1, \dots, B_n\} \backslash Q$ of size $|R| \leq r$, there exist states σ_R^α on R such that

$$\| \mathrm{Tr}_{\bar{R}} N(\rho_A) - \sum_\alpha \mathrm{Tr}(M_\alpha \rho_A) \sigma_R^\alpha \|_1 \leq \epsilon$$

(measure-and-prepare)

for all ρ_A on A.

Note $\{M_\alpha\}$ chosen independent of R.



Excluded region: "Quantum Markov blanket"

On all regions R away from Q, evolution $A \to R$ is approx. measure-and-prepare.

$|Q| = O(1)$, doesn't scale with total system size!

Proof ideas

Theorem statement (for states)

Consider any quantum state $\rho_{AB_1\ldots B_n}$ on $A \otimes B_1 \otimes \ldots \otimes B_n$.

Theorem: For any any $\epsilon > 0$ and $r \in \{1, \ldots, n\}$, there exist states ρ_α^A, probabilities p_α, and an "excluded" subset $Q \subset \{B_1, \ldots, B_n\}$ of size

$$|Q| \leq \frac{1}{\epsilon^2} 2r \ln d_A$$

$d_A = \dim(A)$

such that:

For all subsets R $\subset \{B_1, \ldots, B_n\}$\Q of size $|R| \leq r$,

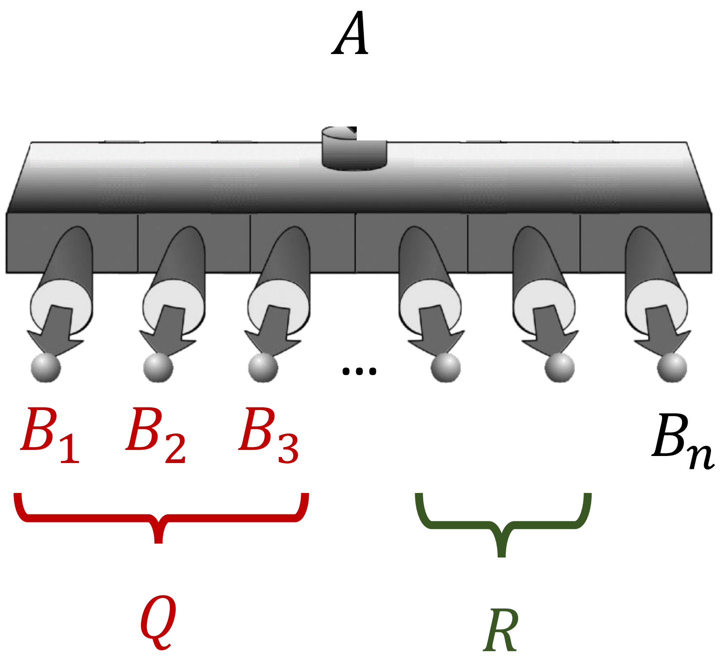
$$\left|\left|\rho_{AR} - \sum_\alpha p_\alpha \rho_\alpha^A \otimes \sigma_\alpha^R\right|\right|_{LOCC_\leftarrow} \leq \epsilon$$

for some choice of states states σ_R^α on R.

Note ρ_α^A, p_α chosen independently of R.

We used "one-way LOCC norm,"

$$||\rho_{AR}||_{LOCC_\leftarrow} \equiv \max_{M_R \in QC} ||(1 \otimes M_R)(\rho_{AR})||_1$$



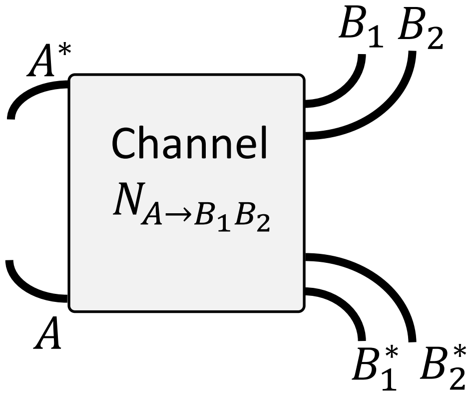
Excluded region: "Quantum Markov blanket"

On all regions R away from Q, state on AR is approx. separable.

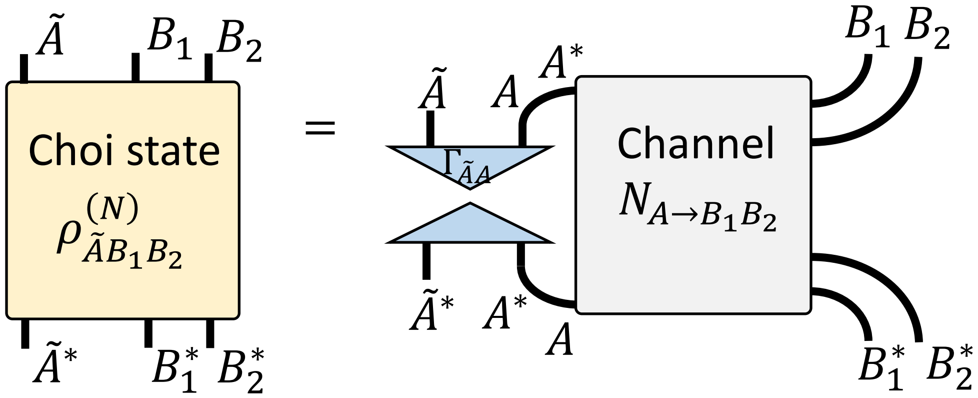
$|Q| = O(1)$, doesn't scale with total system size!

Channel-state duality

The main result may be formulated as a result about either (1) channels with multiple outputs, or (2) multipartite states.



Channel-state duality



imply

Constraints on **dynamical** properties of channels (e.g. no cloning)

Constraints on **static** correlation properties of states (e.g. monogamy)

Reduced channels are measure-and-prepare?

Reduced states are separable?

Sketch of argument: Warm-up result

$S(X) = -Tr(\rho_X \log \rho_X)$ (entropy)

(how much there is to know about X)

$I(X,Y) = S(X) + S(Y) - S(XY)$ (mutual information)

(how much knowing Y tells you about X)

$I(X,Y|Z) = I(X,YZ) - I(X,Z)$ (conditional mutual information)

(how much more knowing YZ tells you about X than just knowing Y)

For any state $\rho_{AB_1 \ldots B_n}$, for any size q:

There exists region $Q \subset \{B_1, \ldots, B_n\}$ of size $|Q| \leq q$ such that for all $B_i \notin Q$,

$$I(A, B_i|Q) \leq \frac{1}{|Q|} 2 \log(d_A).$$

$I(X,Y|Z) = I(X,YZ) - I(X,Z)$ (conditional mutual information)

For any state $\rho_{AB_1 \ldots B_n}$, for any size q:
There exists region $Q \subset \{B_1, \ldots, B_n\}$ of size $|Q| \leq q$ such that for all $B_i \notin Q$,

$$I(A, B_i|Q) \leq \frac{1}{|Q|} 2\log(d_A).$$

Constructive proof: Build up Q by expanding it one by one.

1) Choose region B_{i_1} that maximizes $I(A, B_{i_1})$
2) Choose region B_{i_2} that maximizes $I(A, B_{i_2}|B_{i_1})$
3) Choose region B_{i_3} that maximizes $I(A, B_{i_3}|B_{i_1}B_{i_2})$

$\vdots$

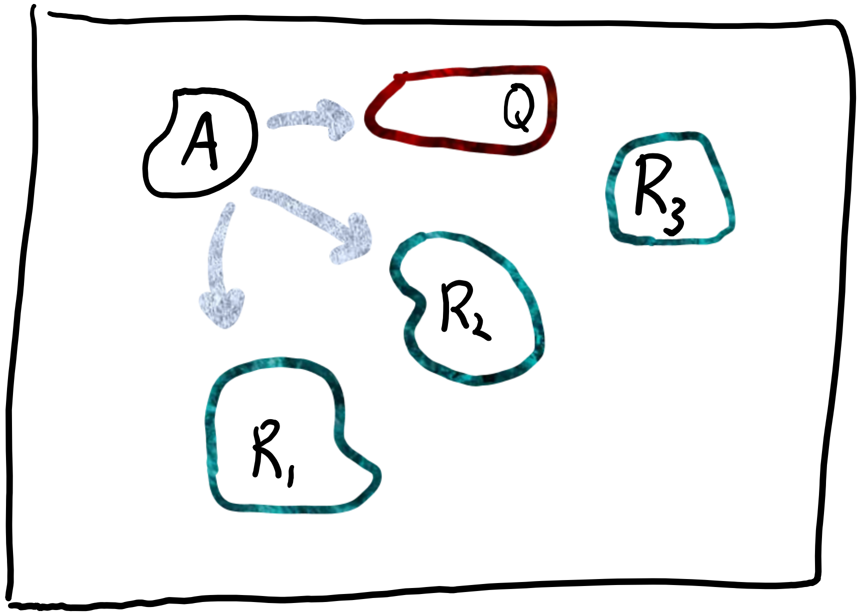
$|Q|$) Choose region $B_{i_{|Q|}}$ that maximizes $I(A, B_{i_q}|B_{i_1} \ldots B_{i_{q-1}})$

By chain rule of mutual information,
$I(A, B_{i_1}) + I(A, B_{i_2}|B_{i_1}) + \ldots + I(A, B_{i_q}|B_{i_1} \ldots B_{i_{q-1}}) = I(A, B_{i_1} \ldots B_{i_q}) \leq 2\log(d_A)$
And by strong subadditivity, all terms are positive. So at least one term is small, i.e. there is some value q' s.t. $I(A, B_{i_{q'}}|B_{i_1} \ldots B_{i_{q'-1}}) \leq |Q|^{-1} 2\log(d_A)$. Take $Q = B_{i_1} \ldots B_{i_{q'-1}}$.

Analyze many-body dynamics?

Constructive method identifies "basis" $\{M^\alpha\}$ on A that is effectively measured/decohered by the rest of the system.

Helps identify emergent classical variables in many-body systems?



Future work

- How tight is the bound? How can it be improved when assuming additional structure to the dynamics, like spatial geometry or local conserved quantities?
- Compatibility theory
- What many-body examples can we explore?
- Use algorithm to identify emergent classical variables in many-body systems?

Thank you!

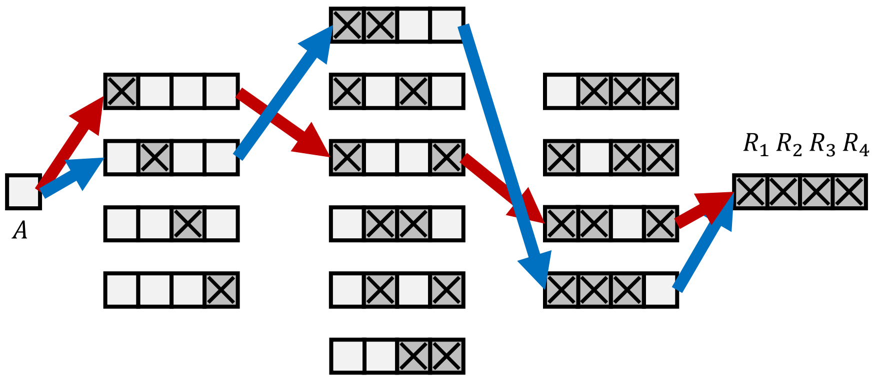
Supplementary slides

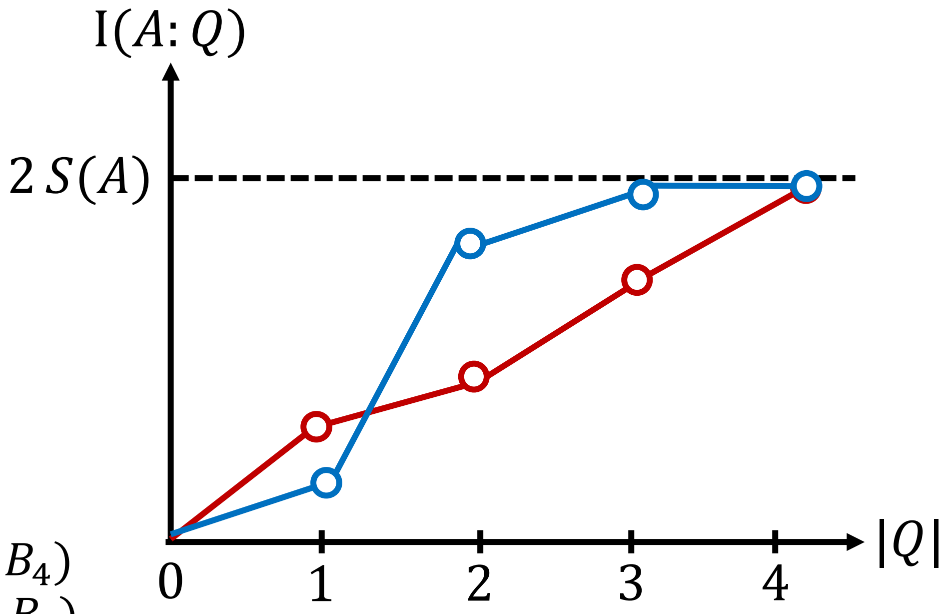
$I(X,Y|Z) = I(X,YZ) - I(X,Z)$ (conditional mutual information)

For any state $\rho_{AB_1\ldots B_n}$, for any size q:
There exists region $Q \subset \{B_1, \ldots, B_n\}$ of size $|Q| \leq q$ such that for all $B_i \notin Q$,

$$I(A, B_i|Q) \leq \frac{1}{|Q|} 2\log(d_A).$$

Visual constructive proof: To find the region Q, optimize over paths below.





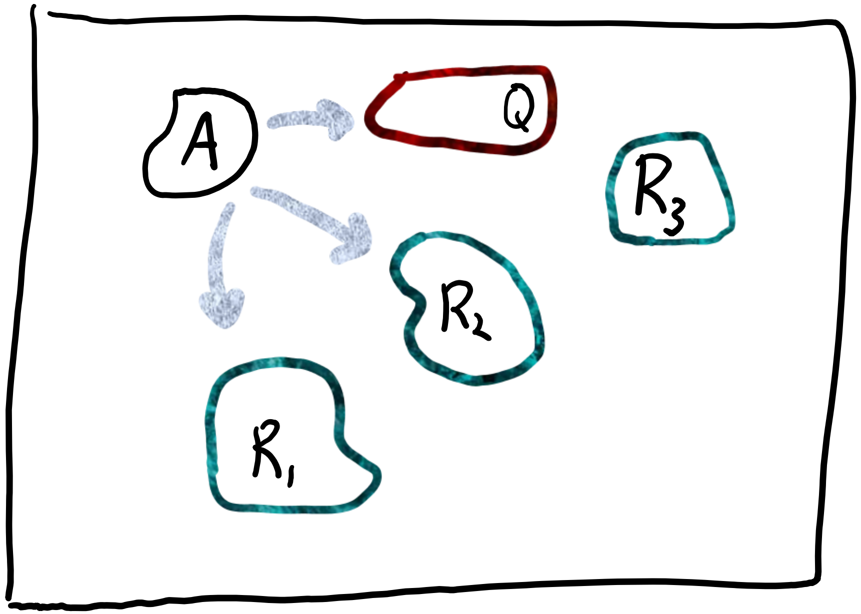
$$I(A{:}B_1) + I(A{:}B_4|B_1) + I(A{:}B_2|B_1B_4) + I(A{:}B_3|B_1B_4B_2) = I(A{:}B_1B_2B_3B_4)$$
$$I(A{:}B_2) + I(A{:}B_1|B_2) + I(A{:}B_3|B_2B_1) + I(A{:}B_4|B_2B_1B_3) = I(A{:}B_1B_2B_3B_4)$$
$$= 2\,S(A)\ \text{(if pure)}$$

Each node is a candidate region Q. Arrows indicate inclusions. Each path is an expanding subset of outputs. Strategy: **Gradually expand Q to learn as much as possible about A, until further expansion yields no further knowledge.** Stop there to obtain final Q.

Analyze many-body dynamics?

Example: Hydrodynamics

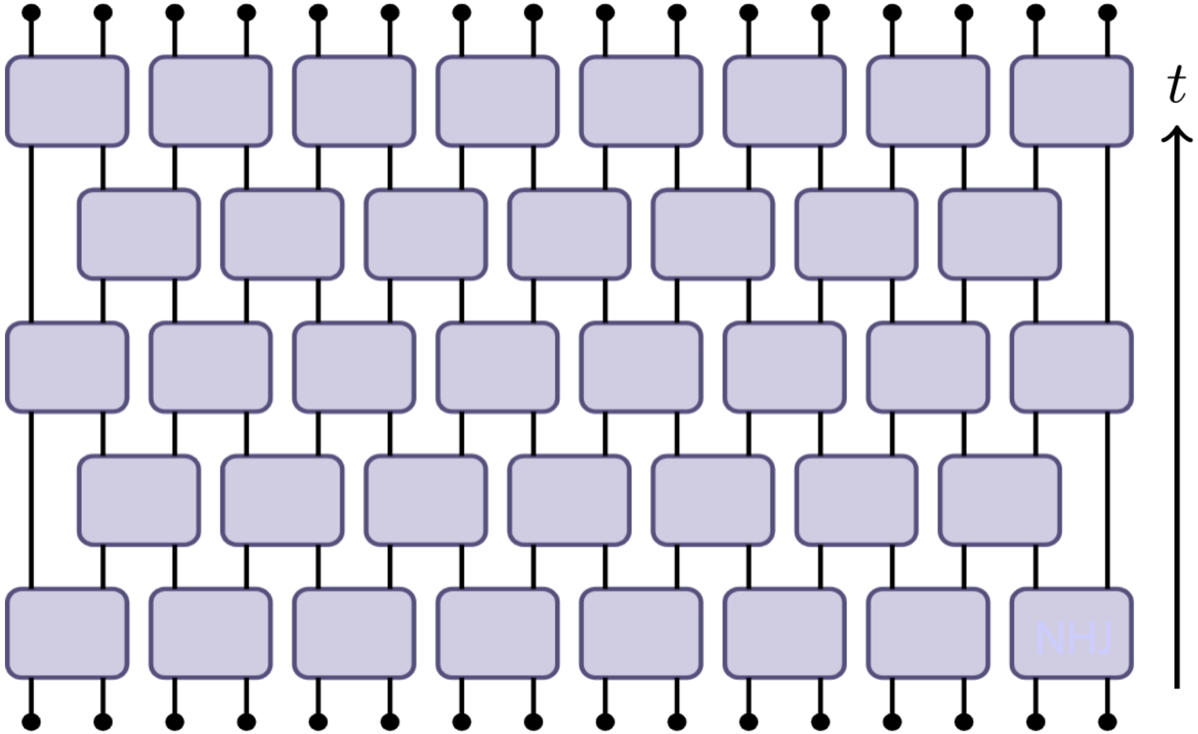
In charge-conserving random circuits, the observable "measured" on A roughly coincides with the charge (confirmed numerically).



Explore more examples?

Inspires simulation method?

Classical simulation of quantum many-body systems



Find which entanglement can be thrown away?
Artificially impose measurements on simulation ("collapse" it) that don't affect local observables?

Example: Spin chain evolution

ρ_A

$\downarrow$ Model evolution as coupling input A to environment B, evolving both, then tracing out all except B_i

$$\rho_{B_i} = Tr_{A\,\overline{B_i}}\left(U_{AB}(\rho_A \otimes \tau_B)U_{AB}^{+}\right)$$

τ_B = Groundstate of spin chain B

U_{AB} = Evolution of extended chain AB

Example:
Couple qubit A onto end of spin chain B, then evolve extended chain

$\rho_A \to \rho_{B_i} =$????

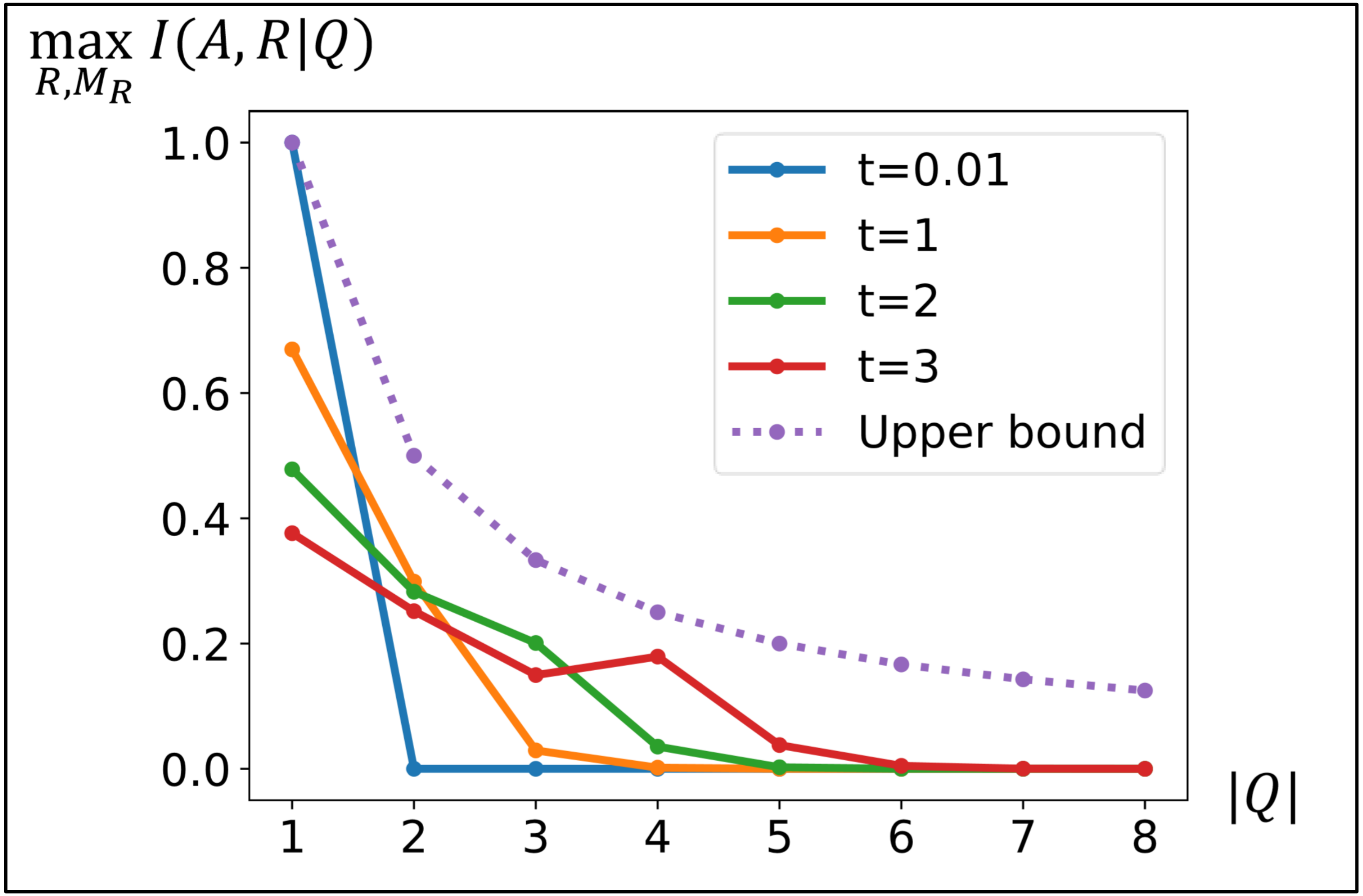
Numerical examples work!

Measure-and-prepare ✓

Numerical example: spin chain

Smaller value means Q is better Markov blanket.

Indicates how close channels $\rho_A \to \rho_R$ must be to m-and-p.



Comparison to de Finetti results; optimality

- Quantum de Finetti results say: marginals of permutation-invariant states are approximately fully separable
- They also say: marginals ρ_{AB_i} of states $\rho_{AB_1 \dots B_n}$ symmetric under permutation of B_i are approximately separable
  1. $||\rho_{AB_i} - \mathrm{Sep}_{\mathrm{AB}}||_1 \leq \frac{2\mathrm{d}^2}{\mathrm{n}}$ (arXiv:0602130, Christandl et al., Theorem II.7')
  2. $||\rho_{AB_i} - \mathrm{Sep}_{\mathrm{AB}}||_{\mathrm{LOCC}_{\leftarrow}} \leq \sqrt{\frac{2\ln(\mathrm{d}_A)}{\mathrm{n}}}$ (arXiv:1010.1750, Brandão, Christandl, Yard)
- Our result gives (1) as special case (but $\mathrm{n} \rightarrow n-1$)

  (Doesn't give their full theorem)
- Exist examples where:

  $||\rho_{AB_i} - \mathrm{Sep}_{\mathrm{AB}}||_1 \geq \frac{d}{2n}\left(1 - \frac{1}{d^2}\right)$ (arXiv:0602130, Christandl et al., Corollary III.9)

One-way LOCC norm

For intermediate results, we use the one-way LOCC norm, defined on bipartite states ρ_{AR},

$$||\rho_{AR} - \sigma_{AR}||_{LOCC_{\leftarrow}} \equiv \max_{M_R \in QC} ||(1 \otimes M_R)(\rho_{AR} - \sigma_{AR})||_1$$

It quantifies the ability to distinguish ρ_{AR} and σ_{AR} using only local operations and one-way communication from $R \rightarrow A$.

It lower bounds the 1-norm as

$$||\rho_{AR} - \sigma_{AR}||_{LOCC_{\leftarrow}} \leq \Omega_{d_A,d_R} ||\rho_{AR} - \sigma_{AR}||_1$$

where

$$\Omega_{d_A,d_R} = \min\{d_A^2, 4d_A^{3/2}, 4d_B^{3/2}, \sqrt{153 d_A d_B}, 2d_B - 1\}$$

(references in paper)