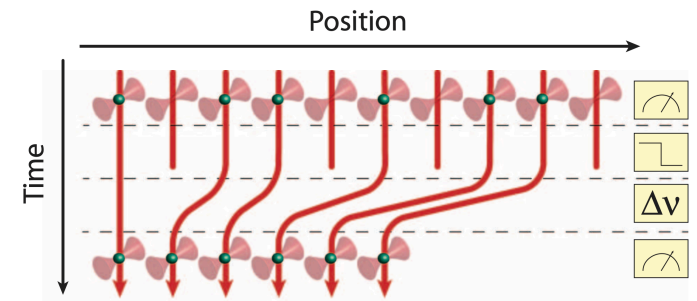
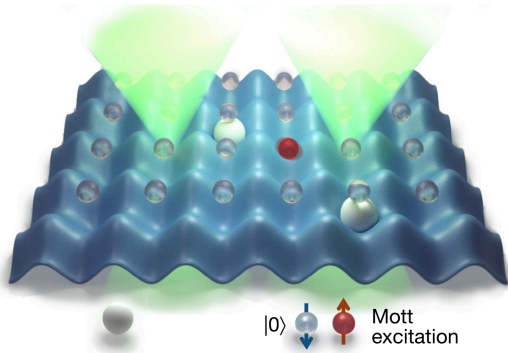


# Strongly Universal Hamiltonian Simulators

Leo Zhou (Harvard University) 

& Dorit Aharonov (Hebrew University) 

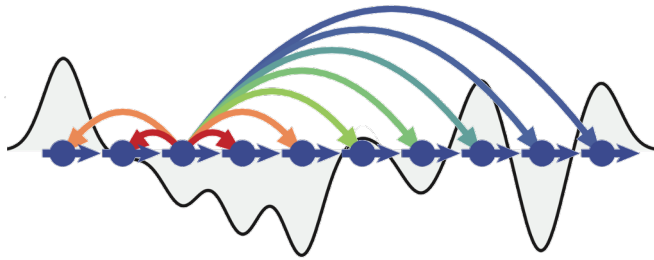
QIP – Feb 5, 2021



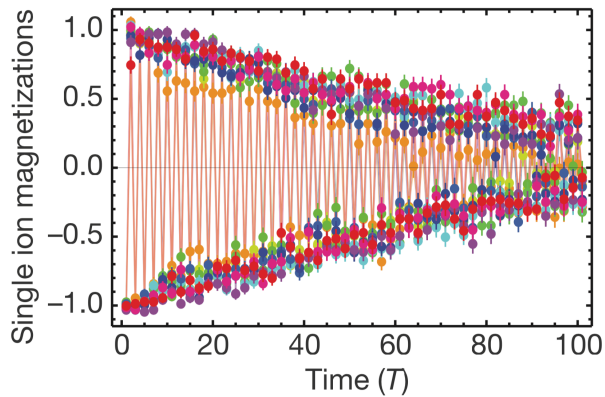
# Analog Quantum Simulation: a promising application

[Feynman 1981] [Cirac Zoller 2012]

## Many-Body Localization & Time Crystals

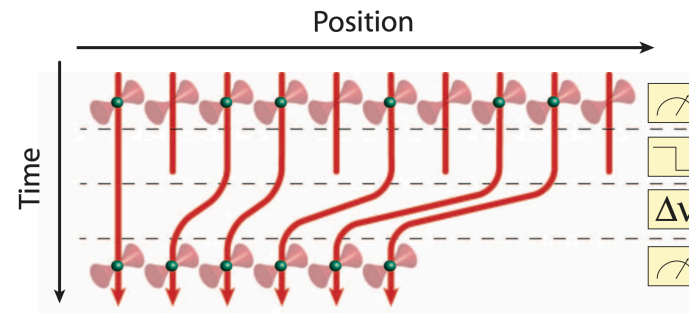


[J. Smith *et al.*, 2016]

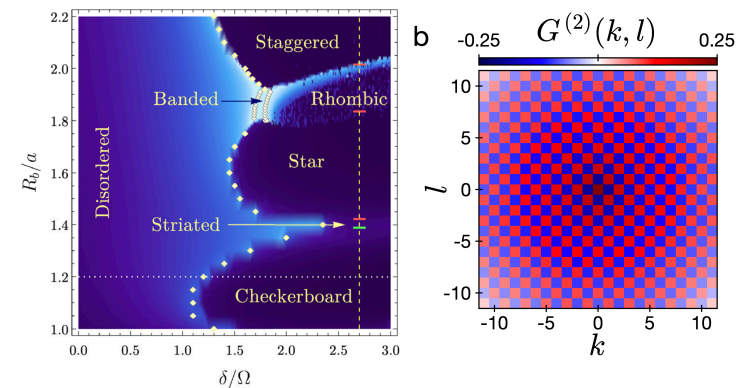


[J. Zhang *et al.*, 2017]

## Quantum Phase Transition

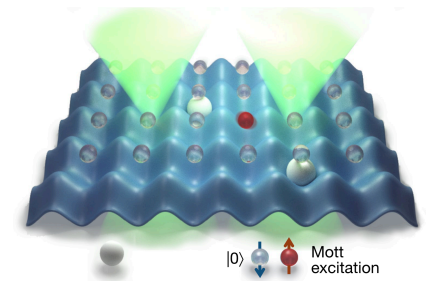


[M. Endres *et al.*, 2016]

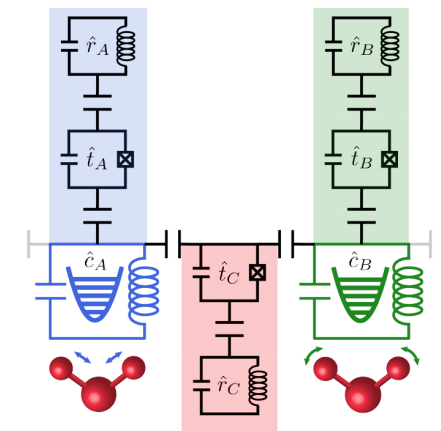


[S. Ebadi *et al.*, 2020]

## Quantum Chemistry



[J. Argüello-Luengo *et al.*, 2018]



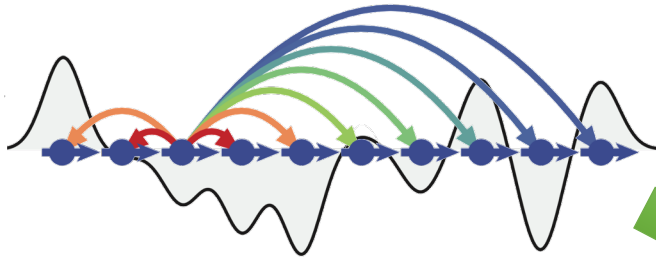
[C. Wang *et al.*, 2019]

# Universal Hamiltonian Simulators

[Cubitt Montanaro Piddock 2017]

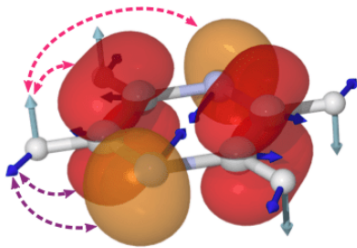
Some families of Hamiltonian are **Universal**:  
*They can simulate all other Hamiltonians*

MBL



$$H = \sum_{i,j} \frac{1}{|i-j|^\alpha} X_i X_j + \sum_i h_i Z_i$$

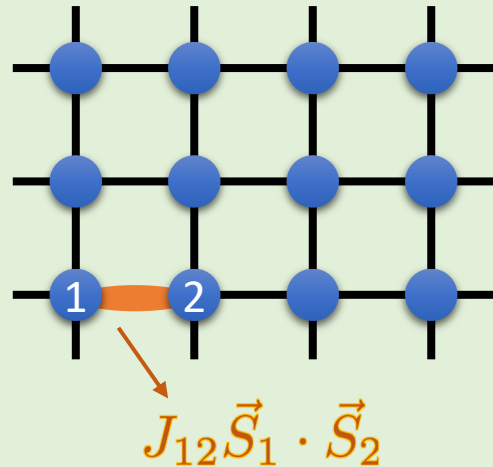
Molecules



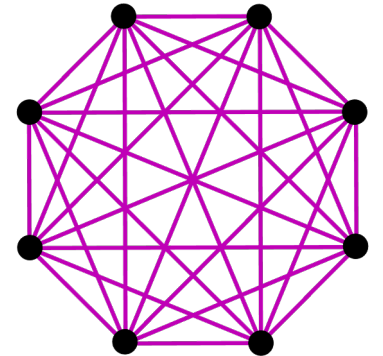
$$H = \sum_{i,j} h_{ij} a_i^\dagger a_j + \sum_{i,j,k,l} V_{ijkl} a_i^\dagger a_j^\dagger a_k a_l$$

Universal Hamiltonians

$$\tilde{H} = \sum_{\langle i,j \rangle \in E} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



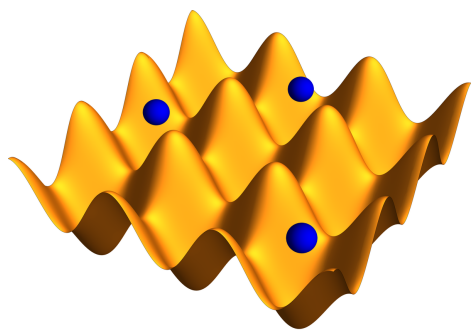
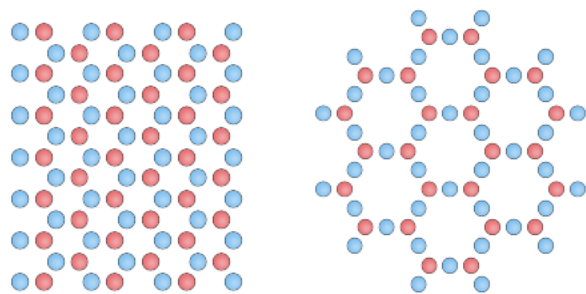
SYK



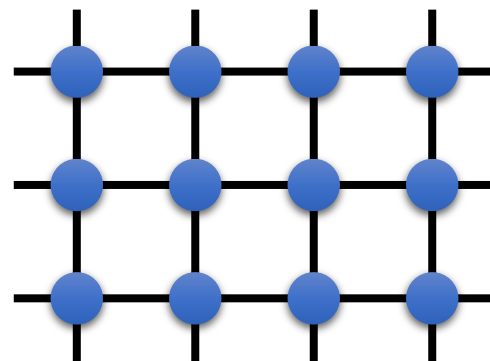
$$H = \sum_{i,j,k,l} J_{ijkl} \gamma_i \gamma_j \gamma_k \gamma_l$$

# What about Resources?

[Cubitt Montanaro Piddock 2017]



$$\tilde{H} = \sum_{\langle i,j \rangle \in E} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



# particles in simulator

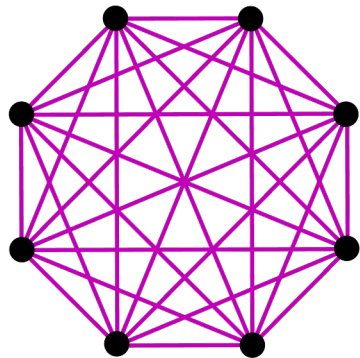
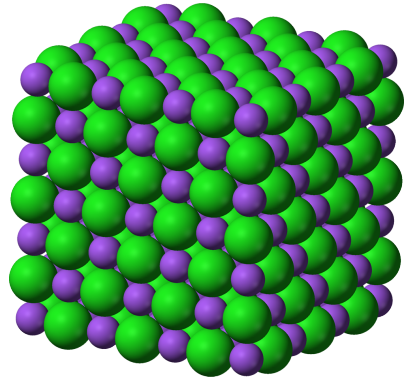
$$\tilde{n} = \text{poly}(n)$$

Interaction energy

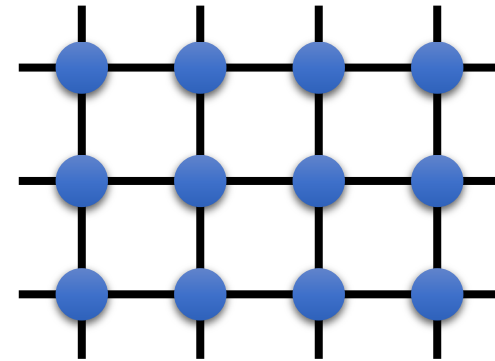
$$J_{ij} = \text{poly}(n)$$

# What about Resources?

[Cubitt Montanaro Piddock 2017]



$$\tilde{H} = \sum_{\langle i,j \rangle \in E} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



# particles in simulator  
 $\tilde{n} = \text{poly}(n)$

Interaction energy

$$J_{ij} = 2^{\text{poly}(n)}$$

**“Weakly Universal”**

as it may require  $\exp(n)$  resources

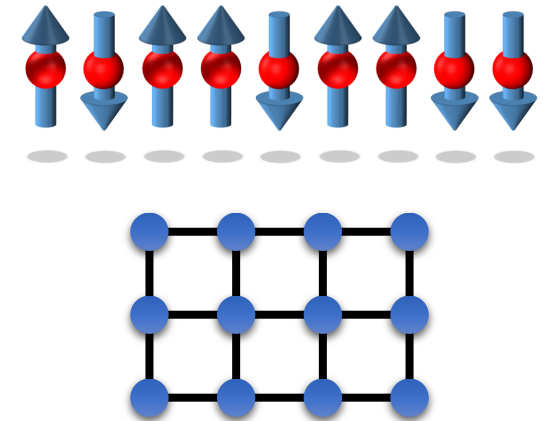
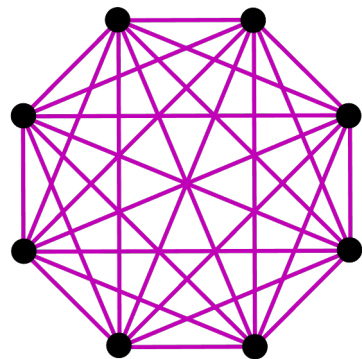
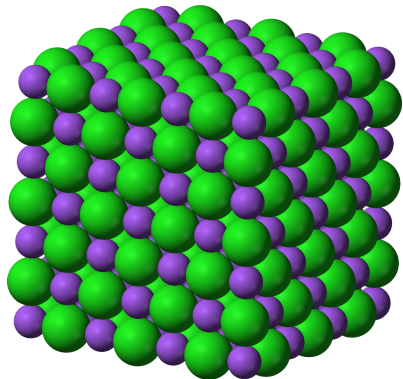
Our Result:

There are simple **Strongly Universal Hamiltonians**

“**Strongly Universal**” = Universal + All resources are **poly( $n$ )**  
for any target Hamiltonian

*vs. **exp( $n$ )** for some target  
Hamiltonian under weak universality*

*Implication: Analog quantum simulation are realistic for general systems*

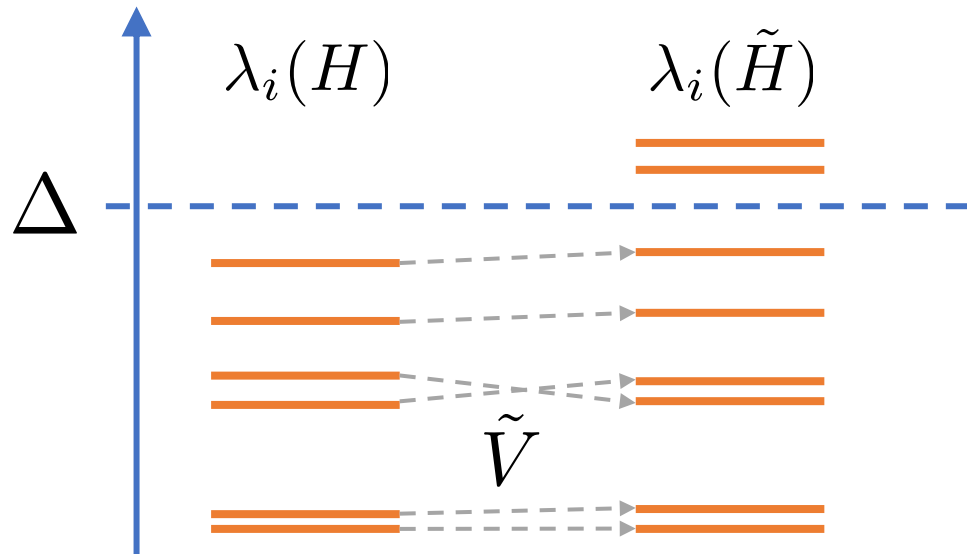


# Overview

- 1) **Defining** Hamiltonian Simulation & Universality
- 2) Review of **Previous** Universal Hamiltonians
- 3) Main Results: **Strongly Universal** Hamiltonians

# Defining Hamiltonian Simulation

We say  $\tilde{H}$   **$(\Delta, \eta, \epsilon)$ -simulates**  $H$  if



$$\| \tilde{H}_{\leq \Delta} - \tilde{V} H \tilde{V}^\dagger \| \leq \epsilon$$

$\tilde{V}$  is an (approximately)  
local isometry encoding

$$\| \tilde{V} - \bigotimes_i V_i \| \leq \eta$$

[Bravyi Hastings 2014] and  
[Cubitt Montanaro Piddock 2017]

# Defining Universality and Translation-Invariance

Consider a family of Hamiltonians

$$\mathcal{F} = \left\{ \tilde{H} = \sum_{\langle i,j \rangle \in E} J_{ij} \hat{h}(\phi_{ij})_{i,j} \right\}$$

operator acting on site  $(i, j)$

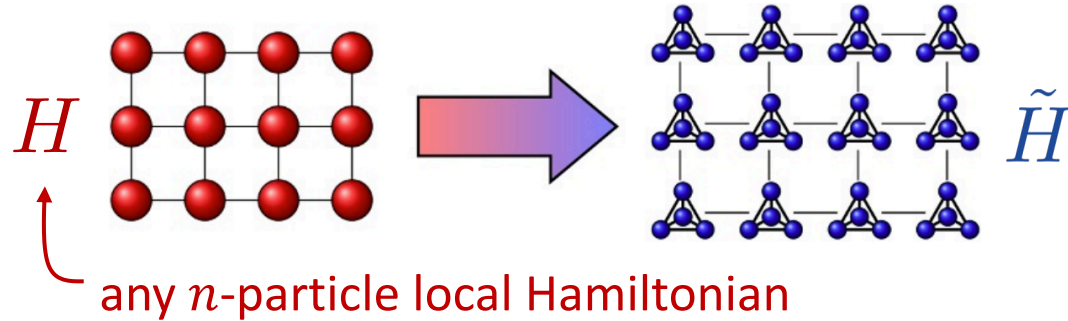
Interaction energy

Site-dependent parameter  $\phi_{ij}$

# Defining Universality and Translation-Invariance

Consider a family of Hamiltonians  $\mathcal{F} = \left\{ \tilde{H} = \sum_{\langle i,j \rangle \in E} J_{ij} \hat{h}(\phi_{ij})_{i,j} \right\}$

## Weak and Strong Universality



$\mathcal{F}$  is **Weakly Universal** if  $\forall H \exists \tilde{H} \in \mathcal{F}$

$\tilde{H}$   $(\Delta, \eta, \epsilon)$ -simulates  $H$

$\mathcal{F}$  is **Strongly Universal** if weakly universal  
and  $\|\tilde{H}\| \leq \text{poly}(n, \Delta, \eta^{-1}, \epsilon^{-1})$

## Semi- and Full-Translation-Invariant (TI)

$\mathcal{F}$  is **Semi-TI** if  $\hat{h}(\phi_{ij}) = \hat{h}$

e.g.

$$\tilde{H} = X_1 X_2 + 3X_2 X_3 + \frac{1}{5} X_3 X_4 + \dots$$

$\mathcal{F}$  is **Full-TI** if semi-TI and  $J_{ij} = J$

e.g.

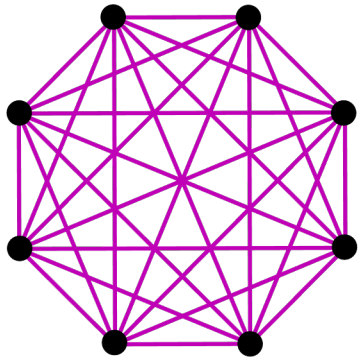
$$\tilde{H} = 12(X_1 X_2 + X_2 X_3 + X_3 X_4 + \dots)$$

# Previous Results: Weak Universality

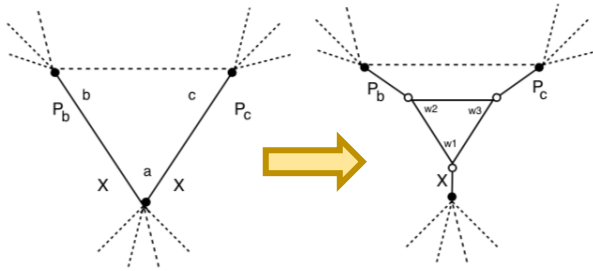
Any  $O(1)$ -local Hamiltonian

e.g. SYK model

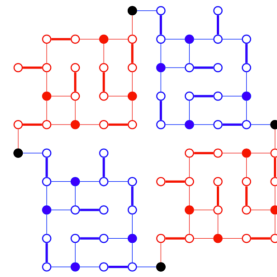
$$H = \sum_{i,j,k,\ell} J_{ijkl} \gamma_i \gamma_j \gamma_k \gamma_\ell$$



(Perturbative) Gadgets



[Oliveira Terhal 2005]



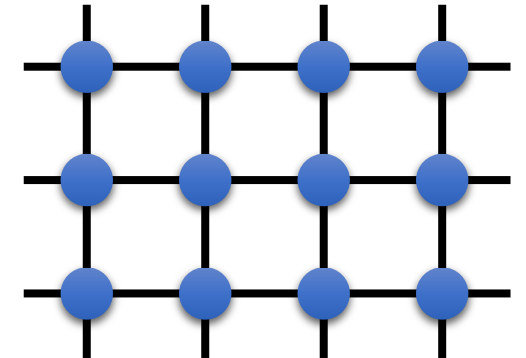
[Piddock Montanaro 2015]



2D, Semi-TI, Weakly Universal Hamiltonians

[Cubitt Piddock Montanaro 2017]

$$\tilde{H} = \sum_{\langle i,j \rangle \in E} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

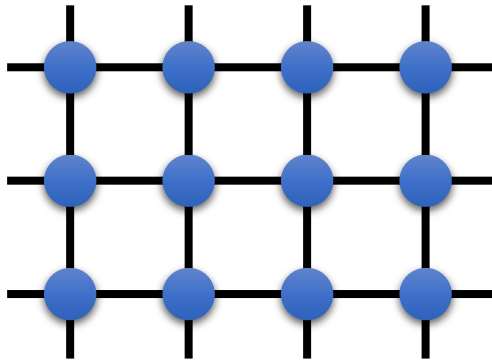


- **Degree-reduction** is necessary to map some Hamiltonians to finite-dimensional lattice
- Previous approach uses  $O(\log n)$  rounds of perturbative gadgets:  $J_{t+1} = [J_t \text{poly}(n)]^c$   
 $\rightarrow$  exponential overhead in energy  $J_{\text{final}} = 2^{\text{poly}(n)}$

# Our Results: Strongly Universal Hamiltonians in 2D and 1D

2D square lattice,  
semi-translation-invariant

$$H = \sum_{\langle i,j \rangle \in E} J_{ij} \hat{h}_{i,j}$$



e.g.

$$\hat{h}_{ij} = X_i X_j + Y_i Y_j \quad \text{or} \quad \vec{S}_i \cdot \vec{S}_j$$

1D, nearest-neighbor, 8-dimensional particles

$$\tilde{H} = \sum_i J_i \hat{h}_{i,i+1} \quad \|\hat{h}_{i,i+1}\| \leq 1$$



$\hat{h}_{i,i+1}$  enforces transition rules  
between configurations

*no TI*

Preprint at [bit.ly/universalHam](https://bit.ly/universalHam)  
(and see arXiv on Monday)

# Our Results vs. Previous Results

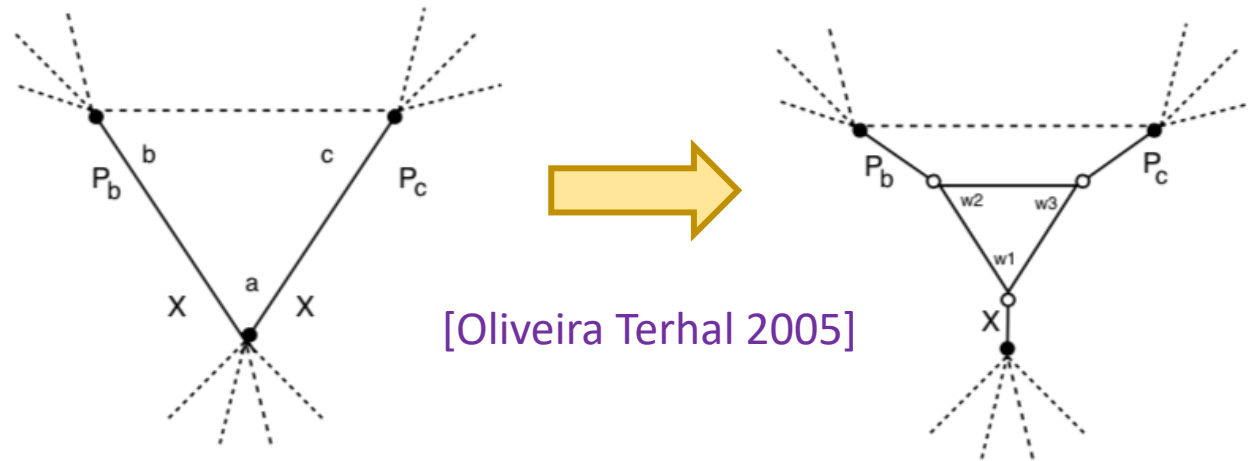
	Spatial dimension	Translation-Invariance	Interaction Energy	Particle Number	
Cubitt <i>et al.</i> (2017)	2D	semi	$\exp(\text{poly}(n))$	$\text{poly}(n)$	} weakly universal
Piddock Bausch (2020)	2D	full	$\exp(\text{poly}(n))$	$\exp(\text{poly}(n))$	
Kohler <i>et al.</i> (2020)	1D	full	$\exp(\text{poly}(n))$	$\exp(\text{poly}(n))$	
Kohler <i>et al.</i> (2020)	1D	full*	$\exp(\text{poly}(n))$	$\text{poly}(n)$	
<b>Our construction</b>	<b>2D</b>	<b>semi</b>	<b><math>\text{poly}(n)</math></b>	<b><math>\text{poly}(n)</math></b>	} strongly universal
<b>Our construction</b>	<b>1D</b>	<b>none</b>	<b><math>\text{poly}(n)</math></b>	<b><math>\text{poly}(n)</math></b>	

- Our results are somewhat ***tight*** since general simulation with  $O(1)$  interaction energy and constant degree is *impossible* [Aharonov Zhou 2018]
- strongly universal  $\subsetneq$  weakly universal (information-theoretical arguments)

# Key Ideas in our construction

## 1) **Non-perturbative** degree-reduction

✗ Degree-reduction with perturbative gadgets requires  $\exp(\text{poly}(n))$  energy



✗ Degree-reduction with  $O(1)$  interaction energy is **impossible** in general

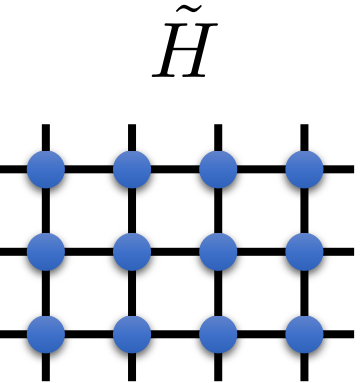
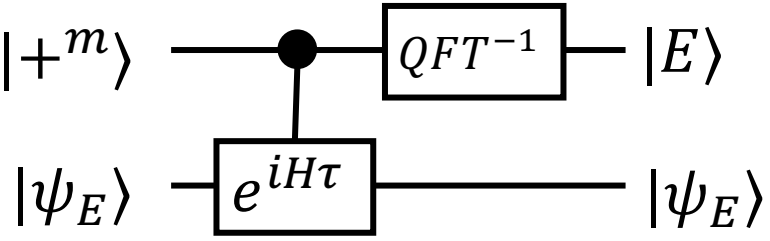
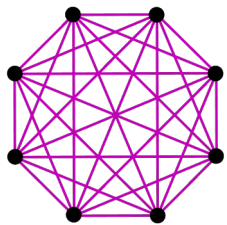
[Aharonov Zhou 2018]

✓ Degree-reduction is **possible** with **poly( $n$ ) interaction energy!** (via circuits)

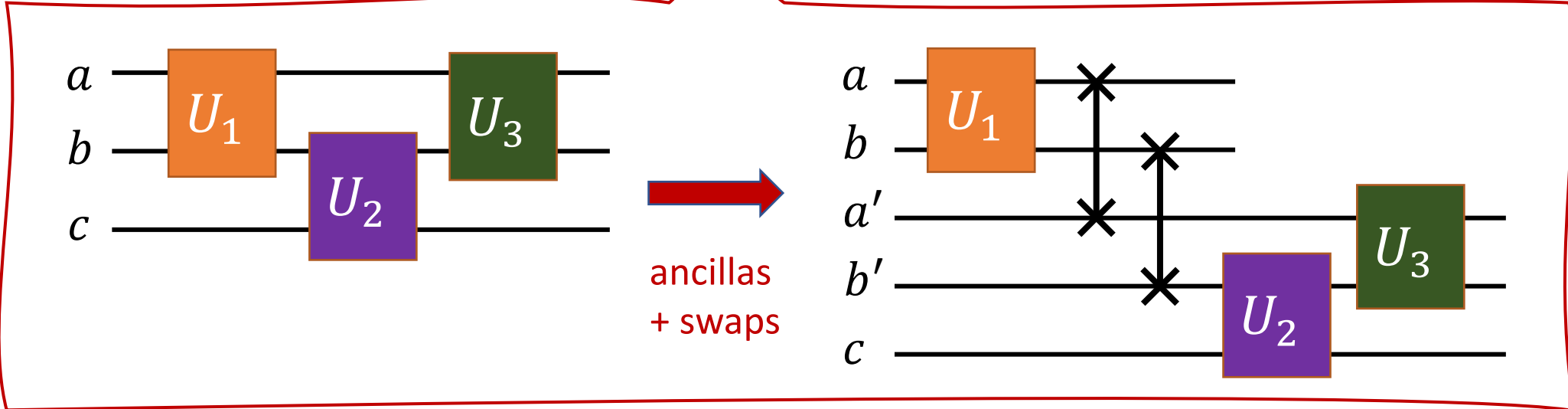
# Key Ideas in our construction

## 1) Non-perturbative degree-reduction (via circuits)

$$H = \sum_E E |\psi_E\rangle \langle \psi_E|$$

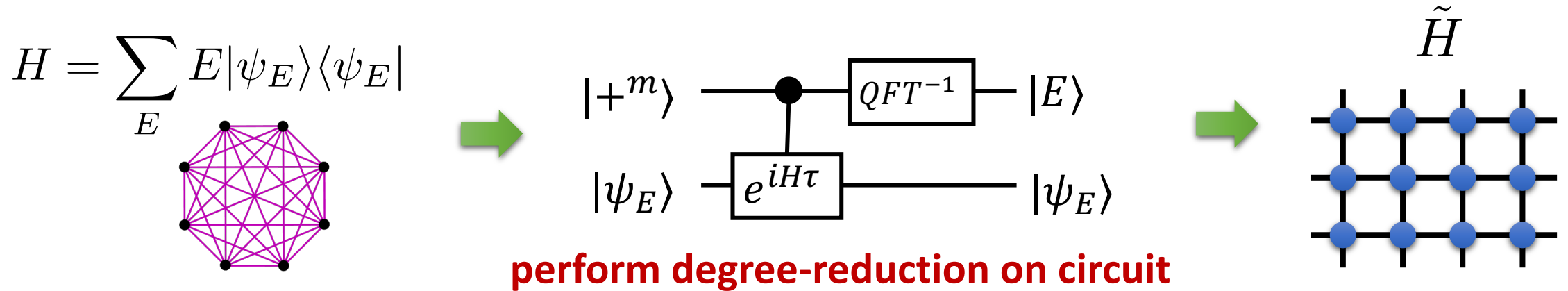


perform degree-reduction on circuit



# Key Ideas in our construction

## 1) **Non-perturbative** degree-reduction (via circuits)



## 2) Recover eigenvalue structure via **bit-wise energy penalty**

$$|E\rangle = |E_1 E_2 \dots E_s \dots\rangle \quad H_{\text{pen}} = \sum_{b=1}^s 2^{-b} |1\rangle\langle 1|_b \otimes P_{\text{clock}}(t = T + b)$$

# Structure of Our Proof / Construction

Any  $O(1)$ -local Hamiltonian



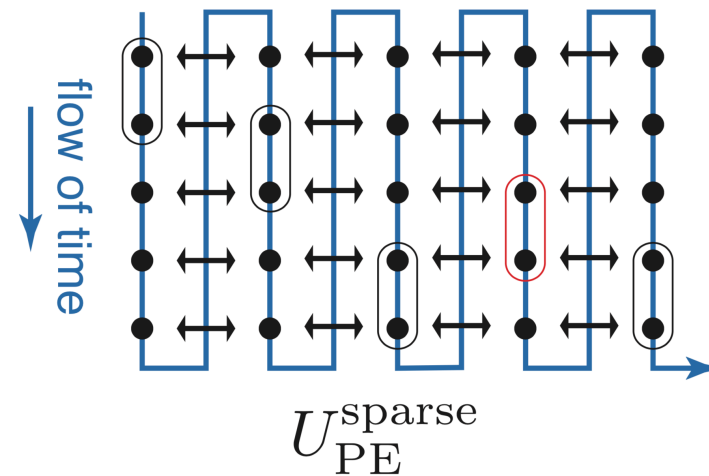
Standard techniques  
(Trotterization ...)

Phase estimation circuit  
using NN gates in 1D



Non-perturbative degree-reduction  
+ bit-wise energy penalty

2D “spatially sparse”  
Hamiltonian



# Structure of Our Proof / Construction

Any  $O(1)$ -local Hamiltonian



Standard techniques  
(Trotterization ...)

Phase estimation circuit  
using NN gates in 1D



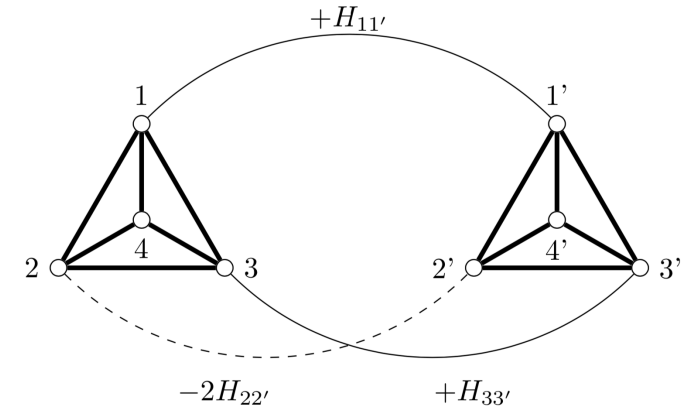
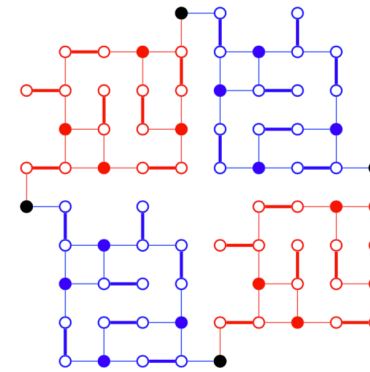
Non-perturbative degree-reduction  
+ bit-wise energy penalty

2D “spatially sparse”  
Hamiltonian



gadgets

2D semi-TI Hamiltonian  
on a square lattice



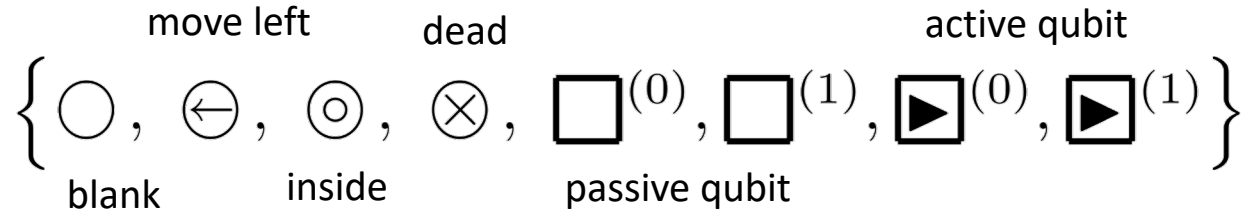
[Oliveira Terhal 2005]

[Pidcock Montanaro 2015]

[Cubitt Montanaro Pidcock 2017]

# Structure of Our Proof / Construction

Any  $O(1)$ -local Hamiltonian



Standard techniques  
(Trotterization ...)

Phase estimation circuit  
using NN gates in 1D

Modified 1D clock  
Hamiltonian +  
bit-wise penalty



[Aharonov et al 2007]  
[Hallgren Nagaj Narayanaswami 2013]

1D NN Hamiltonian on  
8-dimensional particles



Non-perturbative degree-reduction  
+ bit-wise energy penalty

2D “spatially sparse”  
Hamiltonian



gadgets

2D semi-TI Hamiltonian  
on a square lattice

# Summary

- We establish that **strongly** universal analog quantum simulation is possible – can efficiently simulate for **any** target Hamiltonian
  - 1D and 2D universal systems using  $\text{poly}(n)$  qubits and interaction energy
  - Tight since impossible to lower interaction energy to  $O(1)$  [Aharonov Zhou 2018]
- Analog quantum simulation is relevant for many more systems than previously thought

# Open Questions

- 1D semi-TI, strongly universal? Full-TI strongly universal? Fermions?
- Improve the overhead to experimentally relevant regimes?
- Better understand the effects of noise in analog simulation